5. $\frac{x^2-x}{1-x} \cdot \frac{x+2}{x^2-2x} = \frac{x(x-1)}{-1(x-1)} \cdot \frac{(x+2)}{x(x-2)} = \boxed{\frac{(x+2)}{-(x-2)}}$

$x \neq 1, 2$

Extended Practice: Simplify

1. $\frac{2}{x-2} \cdot \frac{x^2-4}{4} = \frac{1}{x-2} \cdot \frac{(x-2)(x+2)}{2} = \boxed{\frac{x+2}{2}}$

$x \neq 2$

2. $\frac{x^2+7x+12}{12} \cdot \frac{4}{x+4} = \frac{(x+4)(x+3)}{3\ 12} \cdot \frac{4}{(x+4)} = \boxed{\frac{x+3}{3}}$

$x \neq -4$

3. $\frac{x^2-64}{x^2-16} \div \frac{x+8}{x+4} = \frac{(x-8)(x+8)}{(x-4)(x+4)} \cdot \frac{(x+4)}{(x+8)} = \boxed{\frac{(x-8)}{(x-4)}}$ $x \neq 4, -4$

4. $\frac{x^2+6x}{6} \cdot \frac{x^2+6}{x^3+6x^2} = \frac{x(x+6)}{6} \cdot \frac{x^2+6}{x^2(x+6)} = \boxed{\frac{x^2+6}{6x}}$ $x \neq 0, -6$

5. $\frac{x^2-4}{2x-4} \cdot \frac{2}{x+2} = \frac{(x-2)(x+2)}{2(x-2)} \cdot \frac{2}{(x+2)} = \boxed{1}$

$x \neq 2$

6. $\frac{x^2-9}{x^2+x} \div \frac{3-x}{x^2-1} = \frac{(x-3)(x+3)}{x(x+1)} \div \frac{-1(x-3)}{(x-1)(x+1)} =$

$= \frac{(x-3)(x+3)}{x(x+1)} \cdot \frac{(x-1)(x+1)}{-1(x-3)} = \boxed{\frac{(x+3)(x-1)}{-x}}$

$x \neq 0, -1, 3$

Sums and Differences of Rational Algebraic Expressions

This section will review how to add and subtract numerical fractions, and then extend the same techniques to rational algebraic expressions.

Review: Simplify

$\left(\frac{3}{3}\right)\frac{5}{12} + \frac{7}{18}\left(\frac{2}{2}\right) = \frac{15}{36} + \frac{14}{36} = \frac{29}{36}$

* Find L.C.M for denominator

The same technique works for subtracting fractions.

The same technique also works for rational algebraic expressions.

Break for Practice: Simplify

1. $\frac{5}{6k} - \frac{4}{3k}\left(\frac{2}{2}\right)$

LCM: $6k$

$\frac{5}{6k} - \frac{8}{6k} = \frac{-3}{6k} = \frac{-1}{2k}$

$k \neq 0$

2. $\frac{a-1}{a^2} + \frac{a+1}{2a}$

LCM: $2a^2$

$\left(\frac{2}{2}\right)\left(\frac{a-1}{a^2}\right) + \frac{a(a+1)}{2a^2} = \frac{2a-2}{2a^2} + \frac{a^2+a}{2a^2} = \boxed{\frac{a^2+3a-2}{2a^2}}$

$a \neq 0$

3. $\frac{3}{x-1} + \frac{2}{x+1} = \frac{3(x+1)}{(x-1)(x+1)} + \frac{2(x-1)}{(x-1)(x+1)} = \frac{3x+3+2x-2}{(x-1)(x+1)} = \boxed{\frac{5x+1}{(x-1)(x+1)}}$

L.C.M: $(x-1)(x+1)$

$x \neq 1, -1$

4. $\frac{x-1}{x+2} - \frac{x^2-5x-2}{x^2+4x+4} = \frac{(x+2)(x-1)}{(x+2)(x+2)} - \frac{(x^2-5x-2)}{(x+2)(x+2)}$

L.C.M: $(x+2)(x+2)$

$$\frac{x^2 - x + 2x - 2 - x^2 + 5x + 2}{(x+2)(x+2)} = \boxed{\frac{6x}{(x+2)(x+2)}}$$

$x \neq -2, 2$

Extended Practice: Simplify

1. $\frac{2}{2}\left(\frac{t+2}{3}\right) + \frac{t-4}{6}$ $\quad \frac{2t+4}{6} + \frac{t-4}{6} = \frac{3t}{6} = \boxed{\frac{t}{2}}$

2. $\left(\frac{z}{z}\right)\left(\frac{z-1}{z}\right) + \frac{z+1}{z^2} = \frac{z^2 - z + z + 1}{z^2} = \boxed{\frac{z^2+1}{z^2}}$ $\quad z \neq 0$

3. $\frac{2}{2}\left(\frac{x+2}{x^2}\right) + \left(\frac{x-2}{2x}\right)\frac{x}{x} = \frac{2x + 4 + x^2 - 2x}{2x^2} = \boxed{\frac{x^2+4}{2x^2}}$ $\quad x \neq 0$

L.C.M = $2x^2$

4. $\frac{3}{3}\left(\frac{t-4}{2t}\right) - \left(\frac{t-6}{3t}\right)\frac{2}{2} = \frac{3t-12}{6t} - \frac{(2t-12)}{6t} = \frac{3t - 12 - 2t + 12}{6t} = \frac{t}{6t}$

$= \boxed{\frac{1}{6}}$ $t \neq 0$

L.C.M = $6t$

5. $\frac{1}{x+1} + \frac{1}{x-1} = \frac{(x-1)}{(x-1)} \cdot \frac{1}{(x+1)} + \frac{(x+1)}{(x+1)} \cdot \frac{1}{(x-1)} = \frac{x - 1 + x + 1}{(x-1)(x+1)}$

L.C.M: $(x+1)(x-1)$

$$= \boxed{\frac{2x}{(x-1)(x+1)}}$$

$x \neq 1, -1$

6. $\frac{1}{1-2x} - \frac{2}{1-4x^2}$

$= \frac{(1+2x)}{(1+2x)} \cdot \frac{1}{(1-2x)} - \frac{2}{(1-2x)(1+2x)} = \frac{1+2x-2}{(1-2x)(1+2x)}$

L.C.M: $(1-2x)(1+2x)$

$1-2x=0$ $\quad -1 \quad -1$ $\quad \frac{-2x=-1}{-2 \quad -2}$ $\quad x \neq \frac{1}{2}$

$1+2x=0$ $\quad -1 \quad -1$ $\quad \frac{2x=-1}{2 \quad 2}$ $\quad x \neq -\frac{1}{2}$

* opposites

$= \frac{*(2x-1)}{*(1-2x)(1+2x)} = \frac{-1}{1+2x}$

$x \neq \frac{1}{2}, -\frac{1}{2}$

7. $\left(\frac{x+2}{x+2}\right) \frac{3}{x^2-5x+6} - \frac{2}{x^2-4} \frac{(x-3)}{(x-3)}$

$(x-2)(x-3) \quad (x-2)(x+2)$

$= \frac{3x+6-2x+6}{(x-2)(x-3)(x+2)}$

L.C.M: $(x-2)(x-3)(x+2)$

$= \frac{x+12}{(x-2)(x-3)(x+2)}$

$x \neq 2, 3, -2$

Equations with Fractional Coefficients

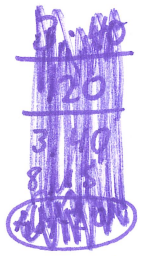
In this section we will use some of what we have learned in this unit to solve equations and inequalities that have fractional coefficients. The key to solving these problems is to clear the fractions first by multiplying through by the least common denominator. (L.C.M)

Break for Practice: Solve

1. $\left(\frac{4t}{3} + \frac{3t}{10} = \frac{7}{5}\right) \cdot 1$ $\rightarrow$ $\frac{4t}{3} \cdot \frac{30}{1} + \frac{3t}{10} \cdot \frac{30}{1} = \frac{7}{5} \cdot \frac{30}{1}$

$40t + 9t = 42 \rightarrow \frac{49t}{49} = \frac{42}{49}$

$t = \frac{6}{7}$



2. $\left(\frac{3r-4}{5} - \frac{2r+1}{4} = -\frac{1}{2}\right) \frac{20}{1}$

$4(3r-4) - 5(2r+1) = -10$

$12r - 16 - 10r - 5 = -10$

$+10 \qquad +10$

$2r - 11 = 0$

$+11 \quad +11$

$\frac{2r}{2} = \frac{11}{2}$

$r = \frac{11}{2}$ or $5\frac{1}{2}$

3. $\left(\frac{y+3}{2}+\frac{3}{5}\geq\frac{y+1}{10}\right)10$

$5(y+3)+6\geq y+1$

$5y+15+6\geq y+1$

$-y\ -21 \qquad -y\ -21$

$\frac{4y}{4}\geq\frac{-20}{4}$

$y\geq -5$

4. How many liters of pure acid must be added to 5 liters of a solution that is 20% acid to make a solution that is 60% acid?

pure = 100% = 1

20% = .2

60% = .6

100% 20% 60%

$1x+.2(5)=.6(x+5)$

$x+1=.6x+3$

$-.6x+1 \qquad -.6x-1$

$\frac{.4x}{.4}=\frac{2}{.4}$

$x = 5$ L of pure acid

or

%	L	total
1.00	x	1x
.20	5	.2(5)
.60	(x+5)	.6(x+5)

rate x time = work done

5. Pump A can unload the *Lunar Petro* in 30 hours and pump B can unload it in 24 hours. Because of an approaching storm, both pumps were used. How long did they take to empty the ship?

Pump A: 30 hrs = 1 ship

$r_A=\frac{1}{30}$

Pump B: 24 hrs = 1 ship

$r_B=\frac{1}{24}$

Pump A pump B

$\left(\frac{t}{30}+\frac{t}{24}=1\right)120$

$4t+5t=120$

$\frac{9t}{9}=\frac{120}{9}$ $t=13.\overline{3}$ hrs

or $13\frac{1}{3}$ hrs

$t \approx 13$ hrs 20 mins

Extended Practice: Solve

1. $\left(\frac{x}{9}+\frac{1}{6}=\frac{2}{3}\right)\frac{18}{1}$

$2x+3=12$
$-3 \quad -3$
$\frac{2x}{2}=\frac{9}{2}$

$x=\frac{9}{2}$

2. $\left(\frac{3u}{5}-\frac{5}{6}=\frac{u}{10}\right)\frac{30}{1}$

$6(3u)-5\cdot5=3u$
$18u-25=3u$
$-3u \quad +25 \quad -3u+25$

$\frac{15u}{15}=\frac{25}{15}$

$u=\frac{5}{3}$

3. $\left(\frac{s-2}{2}-\frac{s-1}{5}=\frac{1}{4}\right)\frac{20}{1}$

$10(s-2)-4(s-1)=5$
$10s-20-4s+4=5$
$6s-16=5$
$+16 \quad +16$
$\frac{6s}{6}=\frac{21}{6}$

$s=\frac{7}{2}$

4. $\left(\frac{z}{4}-\frac{z-1}{6}\leq\frac{5}{12}\right)12$

$3z-2(z-1)\leq 5$
$3z-2z+2\leq 5$
$-2 \quad -2$

$z\leq 3$

5. $\left(\frac{r-2}{8}<\frac{3r+1}{6}+\frac{1}{3}\right)\frac{24}{1}$

$3(r-2)<4(3r+1)+8$
$3r-6<12r+4+8$
$-3r \quad -12 \quad -3r \quad -12$

$\frac{-18}{9}<\frac{9r}{9}$

$-2<r$ or $r>-2$