6. $\left(\frac{t^2}{6}-\frac{t}{2}-\frac{2}{3}=0\right)6$

$t^2-3t-4=0$

$(t+1)(t-4)=0$

$t+1=0 \quad t-4=0$

$-1\ -1 \quad +4\ +4$

$t=-1, 4$

7. $\left(\frac{t(t-1)}{3}=\frac{t+1}{2}\right)6$

$2t(t-1)=3(t+1)$

$2t^2-2t=3t+3$

$-3t-3 \quad -3t \quad -3$

$2t^2-5t-3=0$

$(2t+1)(t-3)=0$

$2t+1=0 \quad t-3=0$

$-1\ -1 \quad +3\ +3$

$\frac{2t}{2}=\frac{-1}{2}$

$t=\frac{-1}{2}, 3$

8. An old conveyor belt takes 21 hours to move one day's coal output from the mine to a rail line. A new belt can do it in 15 hours. How long does it take when both are used at the same time?

* rate · time = work done

Old belt: 21 hrs = 1 day

$r_{old}=\frac{1}{21}$

New belt: 15 hrs = 1 day

$r_{new}=\frac{1}{15}$

$\left(\frac{t}{21}+\frac{t}{15}=1\right)\frac{105}{1}$

$5t+7t=105$

$\frac{12t}{12}=\frac{105}{12}$

$t=8.75$ hrs

or

$t=8$ hrs 45 min

9. How much pure antifreeze must be added to 12 liters of a 40% solution of antifreeze to obtain a 60% solution?

%	L	= solution
100	x	$1x$
.4	12	$.4(12)$
.6	$x+12$	$.6(x+12)$

$1x+.4(12)=.6(x+12)$

$x+4.8=.6x+7.2$

$-.6x\ -4.8 \quad -.6x\ -4.8$

$\frac{.4x}{.4}=\frac{2.4}{.4}$

$x=6$ L

Solving Fractional Equations.

Closely related to the problems that were solved in the previous section, are fractional equations. These equations include variables in the denominator which means that you have to be alert to extraneous (extra solutions that don't work in the original equation) solutions. The key is to remember that the denominators can never equal zero.

Break for Practice: Solve

1. $\left(\frac{3}{y}-\frac{1}{2y}=\frac{5}{4}\right)4y$ → $3\cdot4-2=5\cdot y$

$12-2=5y$

$\frac{10}{5}=\frac{5y}{5}$

$2=y$

$y\neq0$

excluded values

2. $\left(\frac{12}{n}=\frac{12}{n+1}+1\right)\frac{n(n+1)}{1}$ → $12(n+1)=12n+n(n+1)$

$12n+12=12n+n^2+n$

$-12\quad -12\quad -12n$

$0=n^2+n-12$

$0=(n+4)(n-3)$

$n+4=0$ $\quad n-3=0$

$-4\ -4 \quad +3\ +3$

$n=-4 \quad n=3$

$n+1=0$

$n\neq0,-1$

Excluded values

3. $\left(\frac{7}{x-3}-\frac{3}{x-4}=\frac{1}{2}\right)2(x-3)(x-4)$ → $7(2)(x-4)-3(2)(x-3)=(x-3)(x-4)$

$14x-56-6x+18=x^2-3x-4x+12$

$8x-38=x^2-7x+12$

$-8x+38 \quad -8x+38$

$0=x^2-15x+50$

$0=(x-5)(x-10)$

$x-5=0$ $\quad x-10=0$

$+5\ +5 \quad +10\ +10$

$x=5,10$

$x-3=0$

$+3\ +3$

$x-4=0$

$+4\ +4$

excluded values

$x\neq3,4$

4. $\left(\frac{12}{x^2-4}-\frac{3}{x-2}=-1\right)\frac{(x-2)(x+2)}{1}$ → $12-3(x+2)=-(x^2-4)$

$12-3x-6=-x^2+4$

$+x^2-4 \quad +x-4$

$x^2-3x+2=0$

$(x-2)(x-1)=0$

$(x-2)(x+2)$

$x-2=0$ $\quad x+2=0$

$+2\ +2 \quad -2\ -2$

$x-2=0$ $\quad x-1=0$

$+2\ +2 \quad +1\ +1$

$x=2,1$

$x\neq2,-2$

excluded values

1 rate • time = work done 2

5. One pump can empty the town swimming pool in 7 hours less time than a smaller second pump can. Together they can empty the pool in 12 hours. How much time would it take the larger pump alone to empty it?

Pump 1: $x-7$ $28-7$ $P_1 = 21$ hrs

Pump 2: x $P_2 = 28$ hrs

Pump 1 Pump 2 tank

$$\left(\frac{12}{x-7} + \frac{12}{x} = 1\right) x(x-7)$$

$$12x + 12(x-7) = x(x-7)$$

$$12x + 12x - 84 = x^2 - 7x$$

$-24x$ $+84$ $\quad -24x + 84$

$$0 = x^2 - 31x + 84$$

$$0 = (x-3)(x-28)$$

$0 = x-3 \quad x-28$

$+3 \quad +28$

$x = 3, 28$

$3 - 7 = -4$ ✗

Larger pump would take 21 hrs to empty the pool

84
1, 84
2, 42
4, 21
3, 28

Extended Practice: Solve

1. $\left(\frac{3}{t} - \frac{1}{3t} = \frac{2}{3}\right)\frac{3t}{1}$ → $3(3) - 1 = 2t$

$9 - 1 = 2t$

$\frac{8}{2} = \frac{2t}{2}$

$4 = t$ $\quad t \neq 0$

2. $\left(\frac{1}{x} = \frac{2}{x-3}\right) x(x-3)$ → $x - 3 = 2x$

$-x \quad -x$

$-3 = x$

$x - 3 = 0$

$+3 \quad +3$

$x \neq 0, 3$

3. $\left(\frac{2}{s+3} - \frac{1}{s-3} = 0\right)^{(s+3)(s-3)}$ → $2(s-3) - (s+3) = 0$

$2s - 6 - s - 3 = 0$

$s - 9 = 0$

$+9 \quad +9$

$s = 9$

$s + 3 = 0 \quad s - 3 = 0$

$-3 \quad -3 \quad +3 \quad +3$

$s \neq -3, 3$

4. $\left(\frac{x}{x+3}+\frac{1}{x-3}=1\right)^{(x+3)(x-3)}$ → $x(x-3)+x+3=(x+3)(x-3)$

$x+3=0 \quad x-3=0$

$-3\ -3 \quad +3\ +3$

$x \neq -3, 3$

$x^2-3x+x+3=x^2-9$

$-x^2 \quad -3 \quad -x^2 \quad -3$

$\frac{-2x}{-2}=\frac{-12}{-2}$ → $x=6$

5. $\left(\frac{1}{y-2}+\frac{1}{y+2}=\frac{4}{y^2-4}\right)^{(y-2)(y+2)}$ → $y+2+y-2=4$

$(y-2)(y+2)$

$y-2=0 \quad y+2=0$

$+2\ +2 \quad -2\ -2$

$y \neq 2, -2$

$\frac{2y}{2}=\frac{4}{2}$ $y=2$ but it can't so **No solution**

6. $\left(\frac{3}{x+1}-\frac{1}{x-2}=\frac{1}{x^2-x-2}\right)^{(x-2)(x+1)}$ → $3(x-2)-(x+1)=1$

$(x-2)(x+1)$

$x+1=0 \quad x-2=0$

$-1\ -1 \quad +2\ +2$

$x \neq -1, 2$

$3x-6-x-1=1$

$2x-7=1$

$+7 \quad +7$

$\frac{2x}{2}=\frac{8}{2}$ → $x=4$

7. A town's old street sweeper can clean the streets in 60 hours. The old sweeper together with a new sweeper can clean the streets in 15 hours. How long would it take the new sweeper to do the job alone?

* rate x time = work done

Old: $r \cdot \frac{60 \text{ hrs}}{60} = \frac{\text{streets}}{60}$ $r_{old}=\frac{1}{60}$

New: $\frac{r_{new} \cdot x}{x} = \frac{1}{x}$ $r_{new}=\frac{1}{x}$

$\left(\frac{1}{60}+\frac{1}{x}\right)15=1$

$\left(\frac{15}{60}+\frac{15}{x}=1\right)^{60x}$

$15x+15(60)=60x$

$-15x \quad -15x$

$\frac{900}{45}=\frac{45x}{45}$

$20=x$

or

$\frac{15}{60}+\frac{15}{x}=1$

$\left(\frac{1}{4}+\frac{15}{x}=1\right)^{4x}$

$x+60=4x$

$-x \quad -x$

$\frac{60}{3}=\frac{3x}{3}$ →

20 hrs