

Extended Practice:

1. Find the position, n , of the underlined term in each geometric sequence.

a) 5, -25, 125, -625, ..., 3125, ...

$$t_1 = 5$$

$$r = -5$$

$$t_n = 3125$$

$$3125 = \frac{5(-5)^{n-1}}{5}$$

$$625 = (-5)^{n-1}$$

$$\log_{-5} 625 = n-1$$

3125 is the 5th term

b) 27, 9, 3, ..., $\frac{1}{81}$, ...

$$t_1 = 27$$

$$r = \frac{1}{3}$$

$$t_n = \frac{1}{81}$$

$$\frac{1}{81} = \frac{27(\frac{1}{3})^{n-1}}{27}$$

$$0.000457247 = (\frac{1}{3})^{n-1}$$

$$\log_{\frac{1}{3}} 0.000457247 = n-1$$

$$\frac{\log 0.000457247}{\log \frac{1}{3}} = n-1$$

$$7 = n-1$$

$$+1$$

$$8 = n$$

$\frac{1}{81}$ is the 8th term

2. Find the stated number of geometric means between the two given terms.

a) One between 2 and 8

$$r = -2$$

$$r = 2$$

$$n = 3$$

$$8 = 2(r)^{3-1}$$

$$\frac{8}{2} = \frac{2(r)^2}{2}$$

$$\sqrt{4} = \sqrt{r^2}$$

$$\pm 2 = r$$

2, -4, 8 2, 4, 8

b) One between -18 and -36

$$-36 = \frac{-18(r)^{3-1}}{-18}$$

$$\sqrt{2} = \sqrt{r^2}$$

$$\pm \sqrt{2} = r$$

$$r = \sqrt{2}$$

$$r = -\sqrt{2}$$

-18, -18√2, -36
or
-18, 18√2, -36

c) Three between 5 and 80

$$80 = \frac{5(r)^{5-1}}{5}$$

$$\sqrt[4]{16} = \sqrt[4]{r^4}$$

$$\pm 2 = r$$

$$r = 2$$

$$r = -2$$

5, 10, 20, 40, 80 5, -10, 20, -40, 80

d) Two between -4 and 108

$$108 = \frac{-4(r)^{4-1}}{-4}$$

$$\sqrt[3]{-27} = \sqrt[3]{r^3}$$

$$-3 = r$$

-4, 12, -36, 108

3. Tell whether each sequence is arithmetic or geometric. Then find a formula for the sequence.

a) The sequence of positive even integers. $2, 4, 6, 8, 10, \dots$

arithmetic

$$t_1 = 2$$

$$d = 2$$

$$t_n = 2 + (n-1)2 \quad \text{or} \quad t_n = 2n$$

b) $25, 33, 41, 49, \dots$

$\underbrace{\quad}_8 \quad \underbrace{\quad}_8 \quad \underbrace{\quad}_8$

arithmetic

$$t_1 = 25$$

$$d = 8$$

$$t_n = 25 + (n-1)8 \quad \text{or} \quad t_n = 8n + 17$$

c) $200, -100, 50, -25, \dots$

geometric

$$t_1 = 200$$

$$r = -\frac{1}{2}$$

$$t_n = 200\left(-\frac{1}{2}\right)^{n-1}$$

Story Problems with Sequences

In this section we shall see arithmetic and geometric sequences applied to various story problems. You will need to decide if the situation described is arithmetic or geometric in order to solve it.

Break for Practice:

1. A part time teacher takes a position at \$6,600 per year. He receives annual increases of \$250. What will his salary be during his fifteenth year of service?

$6,600, 6,850, 7,100, \dots$

$$t_1 = 6,600$$

$$d = 250$$

$$n = 15$$

$$t_{15} = 6,600 + (15-1)250$$

$$= 6,600 + 14(250)$$

$$t_{15} = \$10,100$$

2. A wealthy man gave his son \$5 on his tenth birthday and decided to double his gift each following year. How much did the boy receive on his 21st birthday?

$$t_1 = 5$$

$$r = 2$$

$$n = 12$$

Term #	1	2	3	...	12
b-day	10	11	12	...	21 st
\$	5	10	20	...	t_{12}

$$t_{12} = 5(2)^{12-1}$$

$$= 5(2)^{11}$$

$$t_{12} = \$10,240$$

3. A new house purchased for \$125,000 is expected to increase in value by 3% per year. What should its value be in 12 years?

$$t_1 = 125,000$$

$$3\% = .03$$

$$r = 1.03$$

$$n = 13$$

term #	1	2	3	...	13
year	0	1	2	...	12
house \$	125,000	128,750			t_{13}

$$125,000 (.03) = 3750$$

$$+ 125,000$$

$$t_{13} = 125,000 (1.03)^{13-1}$$

$$t_{13} = \$178,220.11$$

4. A well drilling firm charges \$0.35 to drill the first foot, \$0.38 for the second foot, and so on in an arithmetic progression. At this rate, how much does the firm charge to drill the last foot of a well 350 feet deep?

$$t_1 = .35$$

$$d = .03$$

$$t_{350} = ?$$

$$n = 350$$

term #	1	2	...	350
foot	1	2	...	350
\$.35	.38	...	t_{350}

$$.35, .38, .41$$

$$t_{350} = .35 + (350 - 1) \cdot .03$$

$$t_{350} = \$10.82$$

for the last foot
in the well

Extended Practice:

1. Allysa has taken a job with a starting salary of \$17,600 and annual raises of \$850. What will be her salary during her fifth year on the job?

$t_1 = 17,600$

arithmetic
 $t_n = t_1 + (n-1)d$

term #	1	2	...	5
year	1	2	...	5
salary	17,600	18,450	...	t_5

$d = 850$

$t_5 = 17,600 + (5-1)850$

$n = 5$
 $t_5 = ?$

$t_5 = \$21,000$

2. Frank has taken a job with a starting salary of \$15,000 and annual raises of 4%. What will be his salary during his third year on the job?

$4\% = .04$
 $r = 1.04$

Geometric

term #	1	2	3
year	1	2	3
salary	15,000	15,600	16,224

$\times 1.04$ $\times 1.04$

year 3: \$16,224

3. An advertisement for a mutual fund claims that people who invested in the fund 5 years ago have doubled their money. If the fund's future performance is similar to its past performance, how much would a \$2,000 investment be worth in 40 years? * took / every 5 yrs to double the \$

geometric

term #	1	2	3	4	5	6	7	8	9
years	0	5	10	15	20	25	30	35	40
\$	2,000	4,000	8,000						

$t_9 = 2000(2)^{9-1}$
 $t_9 = \$512,000$

$n = 9$ $r = 2$ $t_1 = 2,000$

4. A culture of yeast doubles in size every 4 hours. If the yeast population is estimated to be 3 million now, what will it be one day from now?

geometric

$r = 2$

term #	1	2	3	4	5	6	7
# of hours	0	4	8	12	16	20	24
population	3	6	12				t_7

$t_7 = 3(2)^{7-1}$

1 day = 24 hrs

$n = 7$

$t_1 = 3$

$t_7 = 192$ million yeast