Now let's look at a couple of other types of problems that we can solve with the arithmetic sequence formula.

$t_n = t_1 + (n-1)d$

Break for Practice:

1. Find the position, n of the underlined term in each arithmetic sequence.

a) 5, 8, 11, 14, … , <u>68</u> , …

$t_1 = 5 \quad t_n = 68$

$d = 3$

$n = ?$

$t_n = t_1 + (n-1)d$

$68 = 5 + (n-1)3$

$-5 \quad -5$

$\frac{63}{3} = \frac{(n-1)3}{3}$

$21 = n - 1$

$+1 \quad +1$

$n = 22$

$t_{22} = 68$

68 is the 22nd term

b) 88, 83, 78, 73, … , <u>13</u> , …

$t_1 = 88$

$d = -5$

$t_n = 13$

$t_n = t_1 + (n-1)d$

$13 = 88 + (n-1)(-5)$

$-88 \quad -88$

$-75 = (n-1)(-5)$

$\frac{-75}{-5} = \frac{(n-1)(-5)}{-5}$

$15 = n - 1$

$+1 \quad +1$

$16 = n$

$t_{16} = 13$

13 is the 16th term

The other idea we will work on is that of finding arithmetic means.

Arithmetic Means – terms between two given terms in an arithmetic sequence.

2. Find the stated number of arithmetic means between the two given terms.

a) Two between 8 and 35.

$t_1 = 8$

$n = 4$

$t_4 = 35$

$35 = 8 + (4-1)d$

$-8 \quad -8$

$\frac{27}{3} = \frac{3d}{3}$

$d = 9$

$8, \_\_, \_\_, 35$

$t_1 = 8 \qquad t_4 = 35$

8, 17, 26, 35

b) Five between 83 and -25.

$t_1 = 83$

$n = 7$

$t_7 = -25$

$-25 = 83 + (7-1)d$

$-83 \quad -83$

$\frac{-108}{6} = \frac{6d}{6}$

$-18 = d$

$83, \_\_, \_\_, \_\_, \_\_, \_\_, -25$

$t_1 = 83 \qquad t_7 = -25$

83, 65, 47, 29, 11, -7, -25

c) One between -7.8 and 3.6.

$t_1 = -7.8$

$n = 3$

$t_3 = 3.6$

$3.6 = -7.8 + (3-1)d$

$+7.8 \quad +7.8$

$\frac{11.4}{2} = \frac{2d}{2}$

$5.7 = d$

$-7.8, \_\_, 3.6$

$t_1 \qquad t_3$

-7.8, -2.1, 3.6

or Average them

$\frac{-7.8 + 3.6}{2} = -2.1$

Extended Practice:

1. Find the position, n, of the underlined term in each arithmetic sequence.

a) 25, 33, 41, ..., 145 ,...

$t_1 = 25$

$d = 8$

$t_n = 145$

$145 = 25 + (n-1)8$

$-25 \quad -25$

$\frac{120}{8} = \frac{(n-1)8}{8}$

$15 = n - 1$

$+1 \quad +1$

$n = 16$

145 is the 16th term

b) 40, 37, 34, ..., -29, ...

$t_1 = 40$

$d = -3$

$t_n = -29$

$-29 = 40 + (n-1)(-3)$

$-40 \quad -40$

$\frac{-69}{-3} = \frac{(n-1)(-3)}{-3}$

$23 = n - 1$

$+1 \quad +1$

$n = 24$

-29 is the 24th term

2. Find the stated number of arithmetic means between the two given terms.

a) One between -3 and 7

$-3, \_\_, 7$

$\frac{-3+7}{2} = 2$

-3, 2, 7

b) One between 2.3 and 9.1

$2.3, \_\_, 9.1$

$\frac{2.3 + 9.1}{2} = 5.7$

2.3, 5.7, 9.1

c) Two between 15 and 45

$t_1 = 15$

$n = 4$

$t_4 = 45$

$15, \_\_, \_\_, 45$

$t_1 \qquad t_4$

$45 = 15 + (4-1)d$

$-15 \quad -15$

$\frac{30}{3} = \frac{3d}{3}$

$d = 10$

15, 25, 35, 45

d) Four between 15 and 45

$t_1 = 15$

$n = 6$

$t_6 = 45$

$15, \_\_, \_\_, \_\_, \_\_, 45$

$t_1 \qquad t_6$

$45 = 15 + (6-1)d$

$-15 \quad -15$

$\frac{30}{5} = \frac{5d}{5}$

$6 = d$

15, 21, 27, 33, 39, 45

3. How many terms are in the sequence 18, 24, 30, … , 618?

$t_1 = 18$, $d = 6$, $t_n = 618$

$618 = 18 + (n-1)6$

$-18 \quad -18$

$\frac{600}{6} = \frac{(n-1)6}{6}$

$100 = n - 1$

$+1 \quad +1$

$101 = n$

101 terms

4. How many terms are in the sequence 44, 36, 28, … , -380?

$t_1 = 44$, $d = -8$, $t_n = -380$

$-380 = 44 + (n-1)(-8)$

$-44 \quad -44$

$\frac{-424}{-8} = \frac{(n-1)(-8)}{-8}$

$53 = n - 1$

$+1 \quad +1$

$54 = n$

54 terms

Geometric Sequences

Now that we have spent time with arithmetic sequences, we will switch our focus to geometric sequences.

Consider the sequence 3, 6, 12, 24, … Verify this is geometric and identify the common ratio.

$r = 2$

$t_1 = 3$

$t_2 = 6 = 3 \cdot 2 = 3 \cdot 2^1$ (×2)

$t_3 = 12 = 3 \cdot 2 \cdot 2 = 3 \cdot 2^2$ (×2)

$t_4 = 24 = 3 \cdot 2 \cdot 2 \cdot 2 = 3 \cdot 2^3$ (×2)

$t_n = 3 \cdot 2^{(n-1)}$

3 ← first term; 2 ← common ratio

Result: For a geometric sequence, the formula is $t_n = t_1 \cdot r^{(n-1)}$

Break for Practice:

1. Write a formula for each of the following.

a) 1000, 200, 40, 8, …

$t_1 = 1000$, $r = \frac{1}{5}$

$t_n = 1000\left(\frac{1}{5}\right)^{n-1}$

b) -100, 50, -25, 12.5, …

$t_1 = -100$, $r = -\frac{1}{2}$

$t_n = -100\left(-\frac{1}{2}\right)^{(n-1)}$

2. Find the specified term in each geometric sequence.

a) 2048, 1024, 512, ... $t_{20} = \frac{1}{256}$

$t_1 = 2048$

$r = \frac{1}{2}$

$n = 20$

$t_{20} = 2048\left(\frac{1}{2}\right)^{(20-1)}$

$= 2048\left(\frac{1}{2}\right)^{19}$

$t_{20} = \frac{1}{256}$

b) 6, 9, 13.5, ... $t_{10} = 230.66$ or $\frac{59049}{256}$

$t_1 = 6$

$r = \frac{9}{6} = \frac{3}{2}$

$n = 10$

$t_{10} = 6\left(\frac{3}{2}\right)^{10-1}$

$= 6\left(\frac{3}{2}\right)^{9}$

$t_{10} = 230.66$

c) $t_2 = 6$ $t_7 = 192$ $t_{12} = 6144$

$t_2 = 6$

$n = 2$

$t_7 = 192$

$n = 7$

$192 = t_1 \cdot r^{(7-1)}$

$6 = t_1 \cdot r^{(2-1)}$

$\frac{192 = t_1 \cdot r^6}{6 = t_1 r}$

$\sqrt[5]{32} = \sqrt[5]{r^5}$

$2 = r$

$6 = t_1 \cdot 2^{2-1}$

$\frac{6}{2} = \frac{t_1 \cdot 2^1}{2}$

$3 = t_1$

t_{12}

$n = 12$

$t_{12} = 3 \cdot 2^{12-1}$

$= 3 \cdot 2^{11}$

$t_{12} = 6144$

Extended Practice:

1. Write a formula for each of the following.

a) 2, 6, 18, 54, ...

$t_1 = 2$

$r = 3$

$t_n = 2 \cdot 3^{(n-1)}$

b) 500, 100, 20, 4, ...

$t_1 = 500$

$r = \frac{1}{5}$

$t_n = 500\left(\frac{1}{5}\right)^{(n-1)}$

c) 64, -48, 36, -27, ...

$t_1 = 64$

$r = -\frac{48}{64} = \frac{-3}{4}$

$t_n = 64\left(\frac{-3}{4}\right)^{(n-1)}$

2. Find the specified term of each geometric sequence.

a) 2, 6, 18, 54, … $t_{10} = 39366$

$t_1 = 2$ $\quad t_n = 2(3)^{n-1}$

$r = 3$ $\quad t_{10} = 2(3)^{10-1}$

$n = 10$ $\quad t_{10} = 39366$

b) 5, 10, 20, 40, … $t_{12} = 10,240$

$t_1 = 5$ $\quad t_n = 5(2)^{n-1}$

$r = 2$ $\quad t_{12} = 5(2)^{12-1}$

$n = 12$ $\quad t_{12} = 10240$

c) 40, -20, 10, -5, … $t_{11} = \frac{5}{128}$

$t_1 = 40$ $\quad t_n = 40(-\frac{1}{2})^{n-1}$

$r = -\frac{1}{2}$ $\quad t_{11} = 40(-\frac{1}{2})^{11-1}$

$n = 11$ $\quad t_{11} = .0390625 = \frac{5}{128}$

d) -10, 50, -250, 1250, … $t_9 = -3906250$

$t_1 = -10$ $\quad t_n = -10(-5)^{n-1}$

$r = -5$ $\quad t_9 = -10(-5)^{9-1}$

$n = 9$ $\quad t_9 = -3906250$

e) $t_2 = 18$ $\quad t_3 = 12$ $\quad t_5 = \frac{16}{3}$

$n=2$ $\quad n=3$

t_3: $12 = t_1(r)^{3-1}$ $\Rightarrow$ $\frac{12 = t_1(r)^2}{18 = t_1 r^1}$

t_2: $18 = t_1(r)^{2-1}$

$\frac{2}{3} = r$

$18 = t_1(\frac{2}{3})^{2-1}$

$\frac{3}{2}(\frac{18}{1}) = t_1((\frac{2}{3})^1)\frac{3}{2}$

$27 = t_1$

$t_n = 27(\frac{2}{3})^n$

$t_5 = 27(\frac{2}{3})^{5-1}$

$t_5 = \frac{16}{3}$

f) $t_3 = -12$ $\quad t_6 = 96$ $\quad t_9 = -768$

t_6: $96 = t_1 \cdot r^{6-1}$ $\Rightarrow$ $\frac{96 = t_1 \cdot r^5}{-12 = t_1 \cdot r^2}$

t_3: $-12 = t_1 \cdot r^{3-1}$ $\Rightarrow$

$\sqrt[3]{-8} = \sqrt[3]{r^3}$

$-2 = r$

$-12 = t_1(-2)^2$

$\frac{-12}{4} = \frac{4t_1}{4}$

$-3 = t_1$

$t_n = -3(-2)^{n-1}$

$t_9 = -3(-2)^{9-1}$

$t_9 = -768$

Now let's look at a couple of other types of problems that we can solve with the geometric sequence formula.

Break for Practice:

1. Find the position, n, of the underlined term in each geometric sequence.

c) $\frac{1}{9}, \frac{1}{3}, 1, 3, \ldots, \underline{19683}, \ldots$ ($\underline{19683}$ = t_n)

$t_1 = \frac{1}{9}$

$r = 3$

$t_n = 19683$

$9(19683) = \left(\frac{1}{9}(3)^{n-1}\right)9$

$177147 = 3^{n-1}$

$\log_3 177147 = n-1$

$\frac{\log 177147}{\log 3} = n-1$

$11 = n-1$

$+1 \quad +1$

$12 = n$

19683 is the 12th term

d) 17, 34, 68, 136, …, $\underline{34{,}816}$, … ($\underline{34{,}816}$ = t_n)

$t_1 = 17$

$r = 2$

$t_n = 34816$

$\frac{34816}{17} = \frac{17(2)^{n-1}}{17}$

$2048 = 2^{n-1}$

$\log_2 2048 = n-1$

$\frac{\log 2048}{\log 2} = n-1$

$11 = n-1$

$+1 \quad +1$

$12 = n$

34,816 is the 12th term

The other idea we will work on is that of finding geometric means.

Geometric Means – terms between two given terms in a geometric sequence.

2. Find the stated number of arithmetic means between the two given terms.

d) Four between 4 and 972.

$4, \_, \_, \_, \_, 972$ ($4 = t_1$, $972 = t_6$)

$r = 3$

$972 = t_1 \cdot r^{6-1} \Rightarrow 972 = t_1 \cdot r^5$

$t_1 = 4$

$\frac{972}{4} = \frac{4 \cdot r^5}{4}$

$\sqrt[5]{243} = \sqrt[5]{r^5}$

$3 = r$

4, 12, 36, 108, 324, 972

e) Three between 3 and 48.

$3, \_, \_, \_, 48$ ($3 = t_1$, $48 = t_5$)

$r = 2$

$t_1 = 3$

$48 = t_1 \cdot r^{5-1}$

$\frac{48}{3} = \frac{3 \cdot r^4}{3}$

$\sqrt[4]{16} = \sqrt[4]{r^4}$

$\pm 2 = r$

3, 6, 12, 24, 48

and

$r = -2$

3, −6, 12, −24, 48