Extended Practice: Factor Completely

1. $16k^2 - 1$ $(4k-1)(4k+1)$	2. $3z^2 + 4z + 1$ $(3z+1)(z+1)$
3. $5v^2 + 4v - 1$ $(5v-1)(v+1)$	4. $x^2 - xy - 30y^2$ $(x-6y)(x+5y)$
5. $p^2 + 2pq - 24q^2$ $(p+6q)(p-4q)$	6. $16x^2 - 25$ $(4x-5)(4x+5)$
7. $81 - 4h^2$ $(9-2h)(9+2h)$	8. $u^2 - 8uv - 12v^2$ $(u- \quad)(u+ \quad)$ 12: 1, 12 2, 6 3, 4 Prime
9. $4r^2 + 8r + 3$ 4: 1, 4 2, 2 3: 1, 3 $(2r+3)(2r+1)$	10. $6s^2 + st - 5t^2$ 6: 1, 6 2, 3 5: 1, 5 $(6s-5t)(s+t)$
11. $2h^2 + 7hk - 15k^2$ 2: 1, 2 15: 1, 15 3, 5 $(2h-3k)(h+5k)$	12. $36p^2 - 49q^2$ $(6p-7q)(6p+7q)$

More Factoring Polynomials

We saw how to factor the difference of two perfect squares, but what happens if we are working with perfect cubes instead?

To understand how perfect cubes work, we will first look at a multiplication problem. The result will give us clues for factoring perfect cubes.

Multiply: $(x+5)(x^2-5x+25)$

$$x^3 - \cancel{5x^2} + \cancel{25x} + \cancel{5x^2} - \cancel{25x} + 125$$

$$x^3 + 125$$

Result: $\boldsymbol{a^3 + b^3} =$ $(a+b)(a^2-ab+b^2)$ similarly

$\boldsymbol{a^3 - b^3} =$ $(a-b)(a^2+ab+b^2$

Break for Practice: Factor Completely

1. $x^3 + 64$ $a = x$ $b = 4$ $(x+4)(x^2-4x+16)$	2. $2n^3 - 54$ $a = n$ $2(n^3-27)$ $b = 3$ $2(n-3)(n^2+3n+9)$	3. $2x^3 + 250$ $x = a$ $2(x^3+125)$ $5 = b$ $2(x+5)(x^2-5x+25)$

Extended Practice: Factor Completely. (These are a mixture of the different types we have studied.)

1. $x^3 - 8$ $(x-2)(x^2+2x+4)$	2. $x^2 - 16$ $(x-4)(x+4)$
3. $x^2 + 5x - 14$ $(x+7)(x-2)$	4. $64x^3 + 125$ $a = 4x$ $b = 5$ $(4x+5)(16x^2-20x+25)$

5. $3x^2 + 9x - 30$ $3(x^2 + 3x - 10)$ $3(x+5)(x-2)$	6. $x^2 - 9x + 20$ $(x-4)(x-5)$
7. $25x^2 - 9$ $(5x-3)(5x+3)$	8. $2x^2 - 4x - 70$ $2(x^2 - 2x - 35)$ $2(x-7)(x+5)$
9. $8x^3 - 27$ $(2x-3)(4x^2 + 6x + 9)$	10. $4x^2 - 12x + 9$ 4: 1, 4 2, 2 9: 1, 9 3, 3 $(2x-3)(2x-3)$

Factoring by Grouping

One last factoring technique to learn is called factoring by grouping or splitting the middle. This is especially useful for the quadratic polynomials that are difficult to factor by simple inspection.

This technique will be used for polynomials in the form $ax^2 + bx + c$. You need to be able to identify the a, b, and c values.

Example: Factor $9x^2 - 56x + 12$.

a b c

$9x^2 - 2x - 54x + 12$

* have to pull something out

$x(9x-2) - 6(9x-2)$

$(x-6)(9x-2)$

Factors of $a \cdot c =$ 108	Does the sum = b?
1, 108	No
2, 54	yes
3, 36	no
4, 27	no
6, 18	no
9, 12	no

* must be the same

Break for Practice: Factor by splitting the middle.

1. $3x^2 - 16x + 13$

$a \cdot c = 39 : 1, 39$

$3, 13$ ✓

$3x^2 - 3x - 13x + 13$

$3x(x-1) - 13(x-1)$

$\boxed{(3x-13)(x-1)}$

2. $\overset{a}{18}x^2 + \overset{b}{27}x + \overset{c}{10}$

$a \cdot c = 180 : 1, 180$

$2, 90$

$3, 60$

$4, 45$

$5, 36$

$6, 30$

$9, 20$

$12, 15$ ✓

$18x^2 + 12x + 15x + 10$

$6x(3x+2) + 5(3x+2)$

$\boxed{(6x+5)(3x+2)}$

3. $6x^2 + 7x - 10$

$a \cdot c = 60 : 1, 60$

$2, 30$

$3, 20$

$4, 15$

$5, 12$ ✓

$6x^2 - 5x + 12x - 10$

$x(6x-5) + 2(6x-5)$

$\boxed{(x+2)(6x-5)}$

Extended Practice: Factor by splitting the middle

1. $8x^2 - 79x - 10$ $a \cdot c = 80$ $1, 80$ $8x^2 - 80x + x - 10$ $8x(x-10) + 1(x-10)$ $\boxed{(8x+1)(x-10)}$	2. $12x^2 + 25x + 12$ $144: 1, 144$ $12x^2 + 9x + 16x + 12$ $2, 74$ $3x(4x+3) + 4(4x+3)$ $4, 36$ $\boxed{(3x+4)(4x+3)}$ $3, 48$ $6, 24$ $8, 18$ $12, 12$ $9, 16$	3. $18x^2 - 15x + 2$ $a \cdot c = 36$ $18x^2 - 3x - 12x + 2$ $1, 36$ $2, 18$ $3x(6x-1) - 2(6x-1)$ $3, 12$ ✓ $\boxed{(3x-2)(6x-1)}$
4. $4x^2 - 12x + 9$ $a \cdot c = 36$ $4x^2 - 6x - 6x + 9$ $1, 36$ $2, 18$ $2x(2x-3) - 3(2x-3)$ $3, 12$ $4, 9$ $\boxed{(2x-3)(2x-3)}$ $6, 6$ ✓	5. $15x^2 + 8x - 12$ $a \cdot c = 180$ $15x^2 + 18x - 10x - 12$ $1, 180$ $3x(5x+6) - 2(5x+6)$ $2, 90$ $3, 60$ $\boxed{(3x-2)(5x+6)}$ $4, 45$ $5, 36$ $6, 30$ $18, 10$ ✓	I think that 5 problems were enough, don't you?

Solving Polynomial Equations

In this section we will use the factoring techniques you have learned to solve polynomial equations. A key property to remember when solving these is if ab = 0, then a = 0, or b= 0, or both = 0.

Steps for Solving Polynomial Equations:

1. Set the equation equal to zero.
2. Factor the equation.
3. Set each factor equal to zero and solve.

* keep x^2 positive

Break for Practice: Solve each equation. The answers are called "roots", "zeros", or "solutions".

1. $x^2 + 7x = 18$

-18

$x^2+7x-18=0$

$(x+9)(x-2)=0$

$x+9=0$, $-9 \quad -9$, $x=-9$

$x-2=0$, $+2 \quad +2$, $x=2$

$\boxed{x=-9,2}$

2. $3r^2 = 10r + 8$

$-10r-8$

$a \cdot c=24$ 1, 24   2, 12

$3r^2-10r-8=0$

$3r^2-12r+2r-8=0$

$3r(r-4)+2(r-4)=0$

$(3r+2)(r-4)=0$

$3r+2=0$, $-2 \quad -2$, $\frac{3r}{3}=\frac{-2}{3}$, $r=-\frac{2}{3}$

$r-4=0$, $+4 \quad +4$, $r=4$

$\boxed{r=-\frac{2}{3},4}$

3. $x^2 + 25 = 10x$ The solution(s) to this is called a double root. Why?

$-10x \quad -10x$

$x^2-10x+25=0$

$(x-5)(x-5)=0$

$x-5=0$

$x=5$

because they are the same. and it shows up twice.

4. $(a+3)(a-3) = 40$

$a^2 - 3a + 3a - 9 = 40$
$\quad -40 \quad -40$

$a^2 - 49 = 0$

$(a-7)(a+7) = 0$

$a - 7 = 0 \qquad a + 7 = 0$
$+7 \quad +7 \qquad -7 \quad -7$
$a = 7 \qquad a = -7$

$a = -7, 7$

5. $(c-6)^2 = c$

$(c-6)(c-6) = c$

$c^2 - 6c - 6c + 36 = c$
$-c \qquad -c$

$c^2 - 13c + 36 = 0$

$(c-9)(c-4) = 0$

$c - 9 = 0 \qquad c - 4 = 0$
$+9 \quad +9 \qquad +4 \quad +4$
$c = 9 \qquad c = 4$

$c = 4, 9$

Extended Practice: Solve each equation.

1. $(x-1)(x-4) = 0$

$x - 1 = 0 \qquad x - 4 = 0$
$+1 \quad +1 \qquad +4 \quad +4$

$x = 1, 4$

2. $t(t+1)(t-2) = 0$

$(t^2 + t)(t - 2) = 0$

don't need to do this

$t = 0 \qquad t + 1 = 0 \qquad t - 2 = 0$
$-1 \quad -1 \qquad +2 \quad +2$
$t = -1 \qquad t = 2$

$\boxed{t = 0, -1, 2}$

3. $z^2 + 3 = 4z$

$-4z \quad -4z$

$z^2 - 4z + 3 = 0$

$(z-3)(z-1) = 0$

$z - 3 = 0 \qquad z - 1 = 0$
$+3 \quad +3 \qquad +1 \quad +1$
$z = 3 \qquad z = 1$

$\boxed{z = 1, 3}$

4. $x^2 - 12 = 4x$

$-4x \quad -4x$

$x^2 - 4x - 12 = 0$

$(x-6)(x+2) = 0$

$x - 6 = 0 \qquad x + 2 = 0$
$+6 \quad +6 \qquad -2 \quad -2$
$x = 6 \qquad x = -2$

$\boxed{x = -2, 6}$

5. $3r^2 = 4r - 1$

$-4r+1 \quad -4r + 1$

$3r^2 - 4r + 1 = 0$

$(3r-1)(r-1) = 0$

$3r - 1 = 0 \qquad r - 1 = 0$

$+1 \quad +1 \qquad +1 \quad +1$

$\frac{3r}{3} = \frac{1}{3} \qquad r = 1$

$r = \frac{1}{3} \qquad \boxed{r = 1, \frac{1}{3}}$

6. $6x^2 = 1 - x$

$+x-1 \quad -1 \quad +x$

$6x^2 + x - 1 = 0$

$(3x-1)(2x+1) = 0$

$3x - 1 = 0 \qquad 2x + 1 = 0$

$+1 \quad +1 \qquad -1 \quad -1$

$\frac{3x}{3} = \frac{1}{3} \qquad \frac{2x}{2} = \frac{-1}{2}$

$x = \frac{1}{3} \qquad x = -\frac{1}{2} \qquad \boxed{x = \frac{1}{3}, -\frac{1}{2}}$

7. $6(x + 12) = x^2$

$6x + 72 = x^2$

$-6x \ -72 \quad -6x \ -72$

$0 = x^2 - 6x - 72$

$0 = (x-12)(x+6)$

$0 = x - 12 \qquad 0 = x + 6$

$+12 \quad +12 \qquad -6 \quad -6$

$12 = x \qquad -6 = x$

$\boxed{x = -6, 12}$

8. $(u + 3)(u - 3) = 8u$

$u^2 - 3u + 3u - 9 = 8u$

$u^2 - 9 = 8u$

$-8u \quad -8u$

$u^2 - 8u - 9 = 0$

$(u-9)(u+1) = 0$

$u - 9 = 0 \qquad u + 1 = 0$

$+9 \quad +9 \qquad -1 \quad -1$

$u = 9 \qquad u = -1 \qquad \boxed{u = -1, 9}$

9. $3t(t + 1) = 4(t + 1)$

$3t^2 + 3t = 4t + 4$

$-4t \quad -4t \ -4$

-4

$3t^2 - t - 4 = 0$

$(3t-4)(t+1) = 0$

$3t - 4 = 0 \qquad t + 1 = 0$

$+4 \quad +4 \qquad -1 \quad -1$

$\frac{3t}{3} = \frac{4}{3} \qquad t = -1$

$t = \frac{4}{3} \qquad \boxed{t = \frac{4}{3}, -1}$

10. $2(r^2 + 1) = 5r$

$2r^2 + 2 = 5r$

$-5r \quad -5r$

$2r^2 - 5r + 2 = 0$

$(2r-1)(r-2) = 0$

$2r - 1 = 0 \qquad r - 2 = 0$

$+1 \quad +1 \qquad +2 \quad +2$

$\frac{2r}{2} = \frac{1}{2} \qquad r = 2$

$r = \frac{1}{2} \qquad \boxed{r = \frac{1}{2}, 2}$