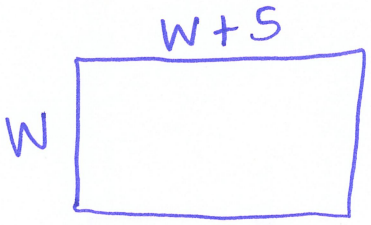
Problem Solving Using Polynomial Equations

Now it's time to see where polynomial equations can be used.

Example: A rectangle is 5 m longer than it is wide, and its area is 176 m^2. Find its dimensions.



$l \cdot w = A$

$w(w+5) = 176$

$w^2 + 5w = 176$

$-176 \quad -176$

$w^2 + 5w - 176 = 0$

$(w-11)(w+16) = 0$

$w - 11 = 0 \qquad w + 16 = 0$

$+11 \quad +11 \qquad -16 \quad -16$

$w = 11 \qquad w = \cancel{-}16$

can't have neg. dimensions

176: 4, 44
8, 22
11, 16 ✓

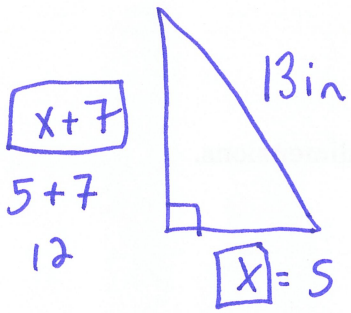
width = 11 m

length = 11+5 = 16 m

11 m x 16 m

Example: The hypotenuse of a right triangle is 13 in long. One leg is 7 in longer than the other leg. Find the length of each leg.

$* \quad a^2 + b^2 = c^2$ Pythagorean Thm



$x^2 + (x+7)^2 = 13^2$

$(x+7)(x+7)$

$x^2 + x^2 + 14x + 49 = 169$

$-169 \quad -169$

$2x^2 + 14x - 120 = 0$

$2(x^2 + 7x - 60) = 0$

$2(x+12)(x-5) = 0$

$x + 12 = 0 \qquad x - 5 = 0$

$x = \cancel{-}12 \qquad x = 5$

60: 2, 30
3, 20
4, 15
5, 12

short leg: 5 in

long leg: 12 in

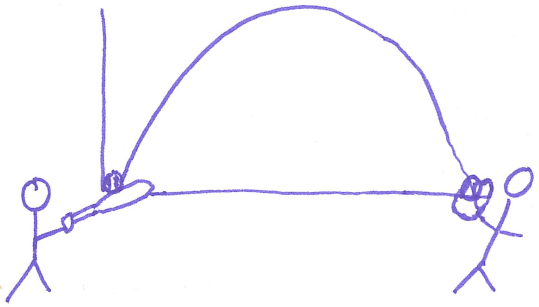
Vertical motion (ex: a thrown ball, a rocket, etc.) affected only by gravity leads to a formula that is a polynomial.

h = height v = initial velocity t = time in seconds

If h is in meters, then $h = vt - 4.9t^2$. If h is in feet, then $h = vt - 16t^2$.

Example: A batter hits a baseball directly upward with a speed of 96 ft/sec.

a) How long is the ball in the air before being caught by the catcher?



$h = vt - 16t^2$

$0 = 96t - 16t^2$

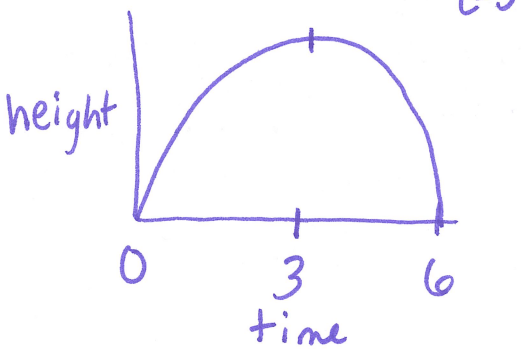
$0 = 16t(6 - t)$

$\frac{0}{16} = \frac{16t}{16}$ $0 = 6 - t$

$0 = t$ $t = 6$

6 sec

b) How high did the ball go?



t = 3

$h = 96(3) - 16(3)^2$

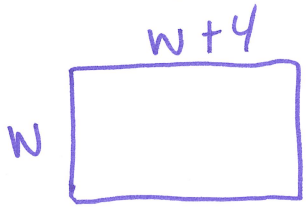
$h = 288 - 16(9)$

$h = 288 - 144$

h = 144 ft

Extended Practice: Solve

1. A rectangle is 4 cm longer than it is wide, and its area is 117 cm². Find its dimensions.



$l \cdot w = A$

$w(w+4) = 117$

$w^2 + 4w = 117$

$-117 \quad -117$

$w^2 + 4w - 117 = 0$

$(w+13)(w-9) = 0$

$w + 13 = 0$ $w - 9 = 0$

$w = -13$ $w = 9$

$w = 9$ length = 9 + 4 = 13

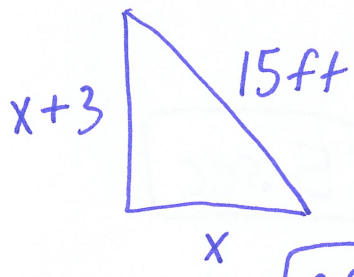
117: 3, 39

9, 13 ✓

9 cm × 13 cm

108: 3, 36; 9, 12

2. The top of a 15-foot ladder is 3 ft. farther up a wall than the foot of the ladder is from the bottom of the wall. How far is the foot of the ladder from the bottom of the wall?



$* a^2+b^2=c^2$

$x^2+(x+3)^2=15^2$

$(x+3)(x+3)$

$x^2+x^2+6x+9=225$

$-225 \quad -225$

$2x^2+6x-216=0$

$2(x^2+3x-108)=0$

$2(x+12)(x-9)=0$

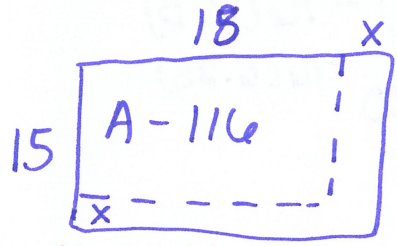
$x+12=0 \quad x-9=0$

$x=\cancel{-12} \quad x=9$

$x=9$

9 ft from the bottom of the wall

3. A rectangle is 15 cm wide and 18 cm long. If both dimensions are decreased by the same amount, the area of the new rectangle formed is 116 cm^2 less than the area of the original. Find the dimensions of the new rectangle.



$l \cdot w = A$

$(18-x)(15-x)=154$

$270-18x-15x+x^2=154$

$x^2-33x+270=154$

$-154 \quad -154$

$x^2-33x+116=0$

$(x-4)(x-29)=0$

$x-4=0 \quad x-29=0$

$x=4, 29$

116: 4, 29

$18-29=-11$ can't be neg.

$x=\cancel{29}$

$15 \cdot 18 = 270$

Orig Area

New Area

$270-116$

$18-4=14$

$15-4=11$

Answer

14 cm x 11 cm

4. A projectile is launched upward from ground level with an initial speed of 98 m/sec.

a) When will it return to the ground?

$h=vt-4.9t^2$

$0=98t-4.9t^2$

$0=4.9t(20-t)$

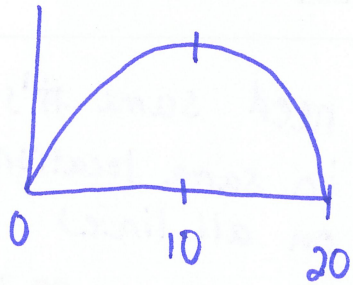
$\frac{0}{4.9}=\frac{4.9t}{4.9} \quad 0=20-t$

$+t \quad +t$

$0=t \quad t=20$

20 sec

b) How high will it go?



$t=10$

$h=98(10)-4.9(10)^2$

$h=980-490$

100

$h=490$ m

5. A ball is thrown directly upward from ground level with an initial speed of 80 ft/sec.

a) When will it return to the ground?

$h = vt - 16t^2$

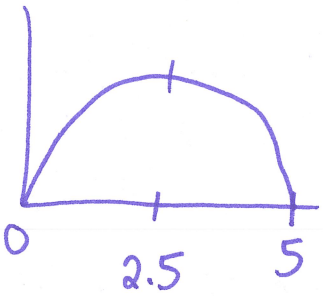
$0 = 80t - 16t^2$

$0 = 16t(5-t)$

$\frac{0}{16} = \frac{16t}{16}$ $t = 0$ $\quad 0 = 5 - t$ $+t \quad +t$ $\quad t = 5$

5 sec

b) How high will it go?



$t = 2.5$ s.

$h = 80(2.5) - 16(2.5)^2$

$h = 200 - 100$ $\quad -16(6.25)$

$h = 100$ ft

Solving Polynomial Inequalities

The final section in this unit is solving polynomial inequalities. The use of factoring and graphing are the keys to solving these problems.

Example: Find and graph the solution set of $x^2 + 3x < -2$.

$x^2 + 3x < -2$

$+2 \quad +2$

① $x^2 + 3x + 2 < 0$

② $(x+2)(x+1) < 0$

③ $x+2 = 0 \quad x+1 = 0$

$x = -2, \quad x = -1$

Steps

1. Set one side of the inequality to zero.
2. Factor
3. Identify the "breaking points" where one factor is 0.
4. Graph separate number lines and combine need total factors +1 lines
5. Identify the solution

④ $(x+2)$ $\quad - - - + + + + + +$ over $-2, -1, 0$

$(x+1)$ $\quad - - - - + + + + +$ over $-2, -1, 0$

$(x+2)(x+1)$ $\quad + \; 0 \; - \; 0 \; + \; 0 \; +$ over $-2, -1, 0$

* need same #'s in same locations on all lines

* ++, -- is where factor is pos/neg

* < 0 we want the solution to be negative ⑤

$-2 < x < -1$

Break for Practice: Find and graph the solution set to each inequality.

1. $x^2 + 15x \leq 4x - 24$

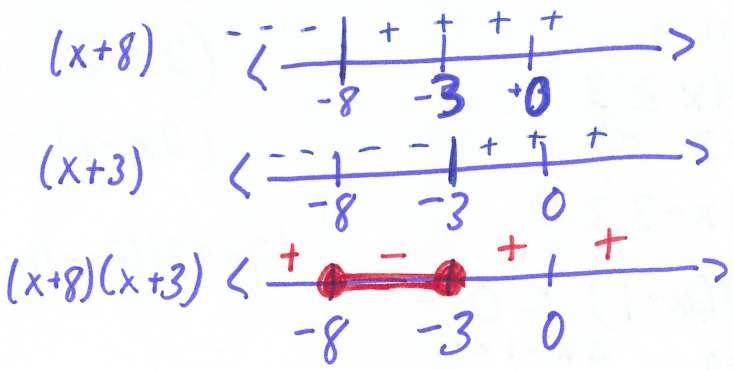
$-4x+24 \quad -4x \quad +24$

$x^2+11x+24 \leq 0$

$(x+8)(x+3) \leq 0$

$x+8=0 \quad x+3=0$

$x=-8 \quad x=-3$



$-8 \leq x \leq -3$

2. $9r(r-1) \geq -2$

$9r^2-9r \geq -2$

$+2 \quad +2$

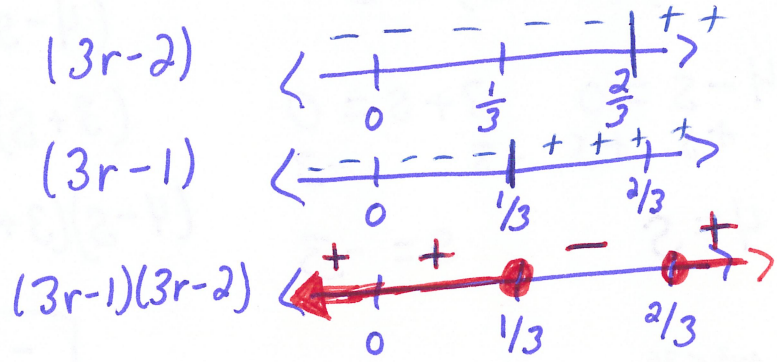
$9r^2-9r+2 \geq 0$

$(3r-2)(3r-1) \geq 0$

$3r-2=0 \quad 3r-1=0$

$+2 \quad +2 \quad +1 \quad +1$

$\frac{3r}{3}=\frac{2}{3} \quad r=\frac{2}{3} \qquad \frac{3r}{3}=\frac{1}{3} \quad r=\frac{1}{3}$



$r \leq \frac{1}{3}$ or $r \geq \frac{2}{3}$

3. $25x - x^3 > 0$

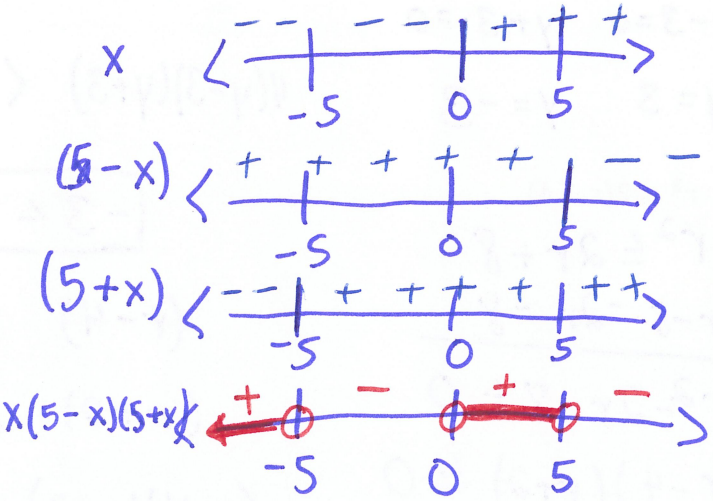
$x(25-x^2) > 0$

$x(5-x)(5+x) > 0$

$x=0 \quad 5-x=0 \quad 5+x=0$

$+x \quad +x \quad -5 \quad -5$

$x=0, \quad x=5, \quad x=-5$



$x<-5$ or $0<x<5$

Extended Practice: Find and graph the solution set to each inequality.

1. $4x(x+1) \geq 3$

$4x^2+4x \geq 3$

$-3 \quad -3$

$4x^2+4x-3 \geq 0$

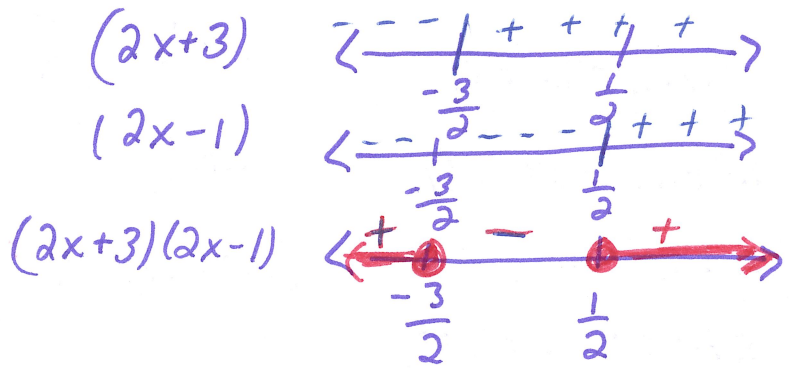
$(2x+3)(2x-1) \geq 0$

$2x+3=0 \qquad 2x-1=0$

$-3 \quad -3 \qquad +1 \quad +1$

$\frac{2x}{2} = \frac{-3}{2} \qquad \frac{2x}{2} = \frac{1}{2}$

$x = -\frac{3}{2} \qquad x = \frac{1}{2}$



$x \leq -\frac{3}{2}$ or $x \geq \frac{1}{2}$

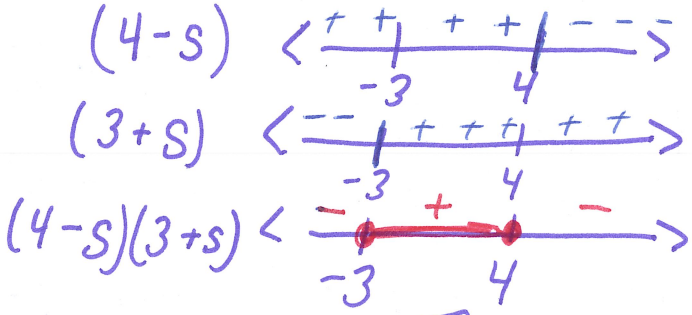
2. $12+s-s^2 \geq 0$

$(4-s)(3+s) \geq 0$

$4-s=0 \qquad 3+s=0$

$+s \quad +s \qquad -3 \quad -3$

$4=s \qquad s=-3$



$-3 \leq s \leq 4$

3. $4y^2 < 36$

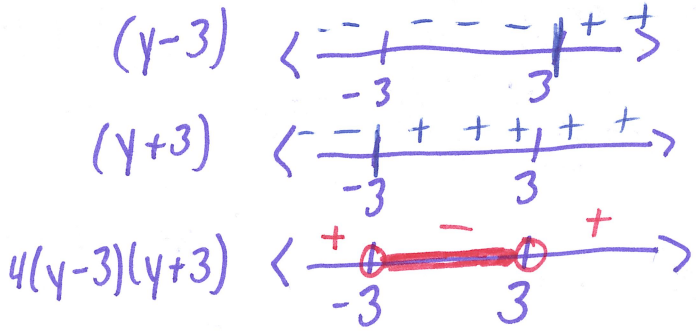
$-36 \quad -36$

$4(y^2-9) < 0$

$4(y-3)(y+3) < 0$

$y-3=0 \qquad y+3=0$

$y=3 \qquad y=-3$



$-3 < x < 3$

4. $r^2 \leq 2(r+4)$

$r^2 \leq 2r+8$

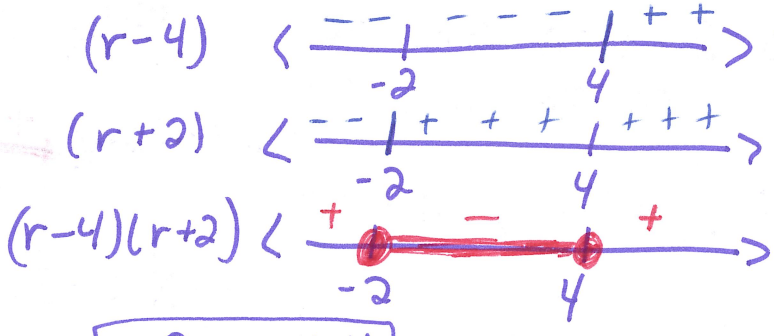
$-2r-8 \quad -2r-8$

$r^2-2r-8 \leq 0$

$(r-4)(r+2) \leq 0$

$r-4=0 \qquad r+2=0$

$r=4 \qquad r=-2$



$-2 \leq r \leq 4$

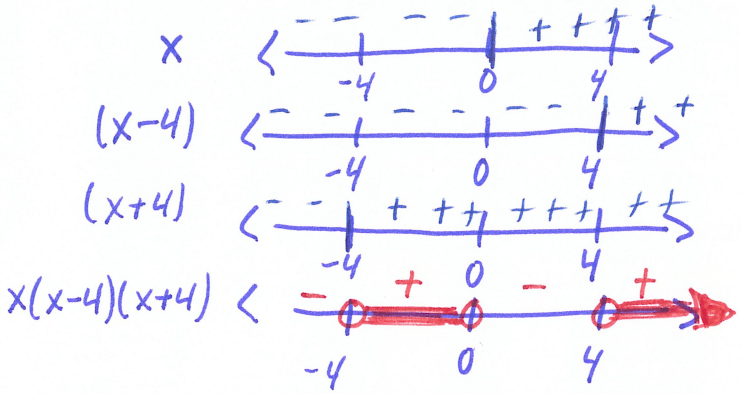
5. $x^3 - 16x > 0$

$x(x^2-16) > 0$

$x(x-4)(x+4) > 0$

$x=0 \quad x-4=0 \quad x+4=0$

$x=0, \quad x=4, \quad x=-4$



$-4 < x < 0 \quad \text{or} \quad x > 4$

6. $y^3 + y^2 < 6y$

$-6y \quad -6y$

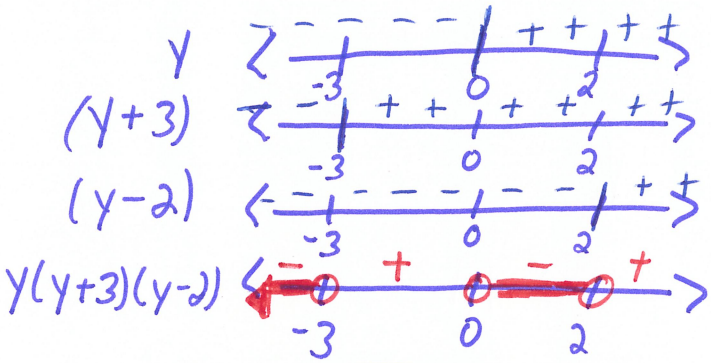
$y^3 + y^2 - 6y < 0$

$y(y^2 + y - 6) < 0$

$y(y+3)(y-2) < 0$

$y=0 \quad y+3=0 \quad y-2=0$

$y=0 \quad y=-3 \quad y=2$



$y < -3 \quad \text{or} \quad 0 < y < 2$

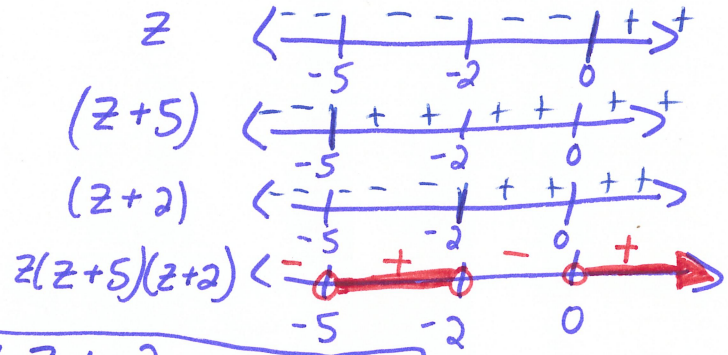
7. $z^3 + 7z^2 + 10z > 0$

$z(z^2+7z+10) > 0$

$z(z+5)(z+2) > 0$

$z=0 \quad z+5=0 \quad z+2=0$

$z=0, \quad z=-5, \quad z=-2$



$-5 < z < -2 \quad \text{or} \quad z > 0$

8. $4z(z-1) \le 15$

$4z^2 - 4z \le 15$

$-15 \quad -15$

4: 1, 4 (2, 2) 15: 1, 15 (3, 5)

$4z^2 - 4z - 15 \le 0$

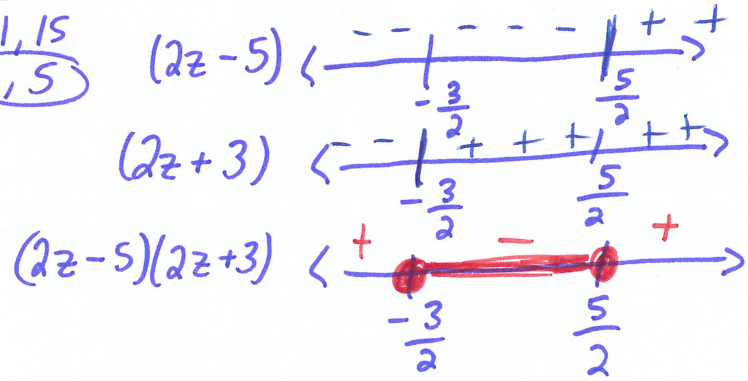
$(2z-5)(2z+3) \le 0$

$2z-5=0 \quad 2z+3=0$

$+5 \quad +5 \quad -3 \quad -3$

$\frac{2z}{2} = \frac{5}{2} \quad \frac{2z}{2} = \frac{-3}{2}$

$z = \frac{5}{2} \quad z = -\frac{3}{2}$



$\frac{-3}{2} \le z \le \frac{5}{2}$