Using Prime Factorization

For the next few sections we will be studying factoring. Before we learn techniques for factoring polynomials, we will work with monomials. In this section, we will concentrate on prime factors, greatest common factors, and least common multiples.

A **factor** is a number that divides into a given number.

Example: List all of the factors of 24. 1, 2, 3, 4, 6, 8, 12, 24

A **prime** is a number whose only factors are one and itself.

Example: List the first ten primes. 2, 3, 5, 7, 11, 13, 17, 19, 23, 29

Greatest Common Factor (GCF): When working with two or more numbers or expressions, the GCF is the largest number/ expression that is a factor of all of the given numbers/expressions.

Example: Find the GCF of 18 and 24. 6
Note: When you are reducing a fraction, you are looking for the GCF.

Least Common Multiple (LCM): When working with two or more numbers or expressions, the LCM is the smallest positive number/expression that has each of the given number/expressions as factors.

Example: Find the LCM of 18 and 24. 72
Note: When you are looking for a common denominator, you are actually looking for the LCM.

One method for finding the GCF or LCM involves using prime factorization. Let's review this.

Example: Write the prime factorizations for 54 and 90.

54
9 6
3 3 2 3

$\boxed{2 \cdot 3^3}$

90
9 10
3 3 2 5

$\boxed{2 \cdot 3^2 \cdot 5}$

Rules for finding GCF and LCM:

1. To find the GCF, take the product of each common factor raised to its lowest power.
2. To find the LCM, take the product of every factor raised to its highest power.

Example: Find the GCF and LCM of 54 and 90.

54: $2 \cdot 3^3$
90: $2 \cdot 3^2 \cdot 5$

GCF: $2 \cdot 3^2 = 2 \cdot 9 = 18$

LCM: $2 \cdot 3^3 \cdot 5 = 2 \cdot 27 \cdot 5 = 270$

Break for Practice: Find the GCF and LCM of each of the following monomials.

1. 60 and 72

60 → 6, 10 → 2, 3, 2, 5

$2^2 \cdot 3 \cdot 5$

72 → 8, 9 → 2, 4, 3, 3 → 2, 2

$2^3 \cdot 3^2$

GCF: $2^2 \cdot 3 = 4 \cdot 3 = 12$

LCM: $2^3 \cdot 3^2 \cdot 5 = 8 \cdot 9 \cdot 5 = 360$

2. 42xy and -70yz

42xy → 6, 7 → 2, 3

$2 \cdot 3 \cdot 7\,xy$

-70yz → 7, 10 → 2, 5

$2 \cdot 5 \cdot 7\,y \cdot z$

GCF: $2 \cdot 7\,y = 14y$

LCM: $2 \cdot 3 \cdot 5 \cdot 7\,xyz$

$210xyz$

3. 10rs, $12rs^4t$, and $14r^3s^2t^2$

10rs → 2, 5

$2 \cdot 5 \cdot r \cdot s$

$12rs^4t$ → 3, 4 → 2, 2

$2^2 \cdot 3\,rs^4t$

$14r^3s^2t^2$ → 2, 7

$2 \cdot 7\,r^3s^2t^2$

GCF: $2rs$

LCM: $2^2 \cdot 3 \cdot 5 \cdot 7\,r^3s^4t^2 = 420r^3s^4t^2$
$4 \cdot 3 \cdot 5 \cdot 7$

Extended Practice: Find the GCF and LCM for each of the following monomials.

1. 45 and 75

45 → 5, 9; 9 → 3, 3
75 → 5, 15; 15 → 3, 5

$3^2\cdot 5$ $\quad 3\cdot 5^2$

GCF = $3\cdot 5$

$\boxed{GCF = 15}$

LCM = $3^2\cdot 5^2$

$9\cdot 25$

$\boxed{LCM = 225}$

2. 315 and -525

315 → 5, 63; 63 → 7, 9; 9 → 3, 3
525 → 25, 21; 25 → 5, 5; 21 → 3, 7

$3^2\cdot 5\cdot 7$ $\quad 3\cdot 5^2\cdot 7$

GCF = $3\cdot 5\cdot 7$

$\boxed{GCF = 105}$

LCM = $3^2\cdot 5^2\cdot 7$

$9\cdot 25\cdot 7$

$\boxed{LCM = 1575}$

3. 30, 35, 36, and 42

30 → 5, 6; 6 → 2, 3
35 → 5, 7
36 → 6, 6; 6 → 2·3, 6 → 2·3
42 → 6, 7; 6 → 2·3

$2\cdot 3\cdot 5$ $\quad 2^2\cdot 3^2$ $\quad 2\cdot 3\cdot 7$

GCF = 1
when nothing else

LCM = $2^2\cdot 3^2\cdot 5\cdot 7$

$4\cdot 9\cdot 5\cdot 7$

LCM = 1260

4. $49x^3$ and $35x^2y$

49 → 7, 7
35 → 5, 7

7^2x^3 $\quad 5\cdot 7x^2y$

GCF = $7\cdot x^2$

$\boxed{GCF = 7x^2}$

LCM = $5\cdot 7^2x^3y$

$\boxed{LCM = 245x^3y}$

5. $52r^2s$ and $78rs^2t$

52 → 4, 13; 4 → 2, 2
78 → 2, 39; 39 → 3, 13

$2^2\cdot 13r^2s$ $\quad 2\cdot 3\cdot 13rs^2t$

GCF - $2\cdot 13rs$

GCF - 26rs

LCM = $2^2\cdot 3\cdot 13r^2s^2t$

$\boxed{LCM = 156r^2s^2t}$

6. $98a^2b^2c$ and $-70abc^2$

98 → 2, 49; 49 → 7, 7
70 → 5, 14; 14 → 2, 7

GCF - $2\cdot 7ab$

GCF = 14ab

$2\cdot 7^2a^2b^2c$ $\quad 2\cdot 5\cdot 7abc^2$ $\quad$ LCM

LCM = $2\cdot 5\cdot 7^2a^2b^2c^2 = 490a^2b^2c^2$

7. $22xy^2z^2$, $33x^2yz^2$, and $44x^2yz$

22 → 2, 11
33 → 3, 11
44 → 4, 11; 4 → 2, 2

$2\cdot 11xy^2z^2$ $\quad 3\cdot 11x^2yz^2$ $\quad 2^2\cdot 11x^2yz$

$\boxed{GCF = 11xyz}$

LCM = $2^2\cdot 3\cdot 11x^2y^2z^2$

LCM = $132x^2y^2z^2$

8. $200a^3b^2c$, $300a^2bc^3$, and $400ab^3c^2$

200 → 10, 20; 10 → 2, 5; 20 → 4, 5; 4 → 2, 2
300 → 10, 30; 10 → 2, 5; 30 → 5, 6; 6 → 2, 3
400 → 10, 40; 10 → 2, 5; 40 → 4, 10; 4 → 2, 2; 10 → 2, 5

$2^3\cdot 5^2a^3b^2c$ $\quad 2^2\cdot 3\cdot 5^2a^2bc^3$ $\quad 2^4\cdot 5^2ab^3c^2$

GCF = $2^2\cdot 5^2abc$

$\boxed{GCF = 100abc}$

LCM = $2^4\cdot 3\cdot 5^2a^3b^3c^3$

$\boxed{1200a^3b^3c^3}$

Factoring Quadratic Polynomials

In this section we will begin reviewing and learning an assortment of methods for factoring polynomials. We will begin with trinomials having a lead coefficient of 1.

Break for Practice: Factor completely. Hint – always look for common factors first.

1. $x^2 + 5x + 6$ $(x+3)(x+2)$	2. $x^2 - 9x + 14$ $(x-2)(x-7)$
3. $x^2 - x - 12$ $(x-4)(x+3)$	4. $x^2 + 7x - 12$ 12: 1, 2, 3, 4, 6, 12 $(x \quad)(x \quad)$ Not factorable Prime
5. $2x^2 + 4x - 48$ $2(x^2+2x-24)$ $2(x+6)(x-4)$	6. $3x^2 - 27x + 24$ $3(x^2-9x+8)$ $3(x-8)(x-1)$

Extended Practice: Factor Completely

1. $x^2 - 9x + 8$ $(x-8)(x-1)$	2. $x^2 + 9x + 14$ $(x+7)(x+2)$

3. $x^2 - 11x + 18$ $(x-9)(x-2)$	4. $x^2 - 10x + 9$ $(x-9)(x-1)$
5. $x^2 + 12x + 20$ $(x+10)(x+2)$	6. $3x^2 - 15x + 18$ $3(x^2-5x+6)$ $3(x-2)(x-3)$
7. $p^2 - 8p + 9$ $(p\quad)(p\quad)$ Prime not factorable	8. $2h^2 - 20h + 48$ $2(h^2-10h+24)$ $2(h-6)(h-4)$
9. $s^2 - 20s + 36$ $(s-18)(s-2)$	10. $z^2 - 9z + 12$ 12: 1, 2, 3, 4, 6, 12 $(z\quad)(z\quad)$ Prime Not factorable
11. $5x^2 + 5x - 60$ $5(x^2+x-12)$ $5(x+4)(x-3)$	12. $t^2 + 2t - 15$ $(t+5)(t-3)$
13. $x^2 - 2x - 35$ $(x-7)(x+5)$	14. $s^2 - 6s - 27$ $(s-9)(s+3)$

Now we need to spend time with some trinomials that are a little more complicated. The same techniques still apply, but you may need more patience since there are more possibilities.

Break for Practice: Factor Completely

1. $6x^2 + 7x - 20$ 6: 1, 6 2, 3 20: 1, 20 2, 10 4, 5 $(3x - 4)(2x + 5)$ 2×4 = 8 3×5 = +15	2. $7x^2 + 2x - 5$ $(7x - 5)(x + 1)$
3. $8x^2 - 22x + 9$ 8: 1, 8 2, 4 9: 1, 9 3, 3 $(2x - 1)(4x - 9)$ 2x9 = 18 1x4 = 4 22	4. $x^2 + 2x + 3$ 3: 1, 3 Not factorable Prime
5. $4x^4 + 10x^3 - 6x^2$ $2x^2(2x^2 + 5x - 3)$ 2: 1, 2 3: 1, 3 $2x^2(2x - 1)(x + 3)$	6. $10x^2 + 7xy + y^2$ $(5x + y)(2x + y)$

Consider the following polynomial. $x^2 - 25$. Is this equivalent to $x^2 + 0x - 25$? Factor it.

Result: **<u>Difference of Two Squares</u>**.

$$a^2 - b^2 = (a + b)(a - b)$$

Break for Practice: Factor Completely

1. $49x^2 - 4$ $(7x - 2)(7x + 2)$	2. $27 - 3h^2$ $3(9 - h^2)$ $3(3 - h)(3 + h)$