Algebra II
Unit 4
Polynomials

Unit "I can" statements:

1. I can simplify, add, subtract, and identify the degree of polynomials.
2. I can use the laws of exponents to multiply a monomial and polynomial.
3. I can multiply polynomials.
4. I can use prime factorization to identify the GCF and LCM of integers and monomials.
5. I can factor polynomials by applying a variety of strategies.
6. I can solve polynomial equations.
7. I can solve applications by using polynomial equations.
8. I can solve polynomial inequalities.

Common Core State Standards that are addressed in this unit include: A.CED.2a, A.CED.3a, A.SSE.1a, A.SSE.1b, A.SSE.2a, F.IF.8
For more information see www.corestandards.org/Math/

Introduction to Polynomials

In this unit we will focus on a special type of expression called a polynomial. We will learn how to simplify, operate with, and factor polynomials. We will also investigate applications.

Definition: Polynomials – expressions that involve only the operations of addition, subtraction, and multiplication on variables. Exponents have to be positive whole numbers. Terms are separated by addition and subtraction.

Break for Practice: Identify which of the following are polynomials. For those that are polynomials, state the number of terms. For those that are not polynomials, explain why.

1. $4x^2 + 5x - 7$ yes, 3 terms
2. $3x - 2$ yes, 2 terms
3. $\frac{2+x}{4x}$ not a polynomial
4. $\frac{4+x}{3} = \frac{4}{3} + \frac{1}{3}x$ yes, 2 terms
5. $7x^2y^5$ yes, 1 term
6. 8 yes, 1 term
7. $\sqrt{7x}$ not a polynomial
8. $\sqrt{7}x$ yes, 1
9. $|x^3 - 2|$ not a polynomial
10. $5x^2y^3 - 2xy^5 - 7$ yes, 3 terms

Definition: Factors – the parts of a term that are multiplied together to form the term.

Example: In the term $7x^4$, there are 5 factors. They are $7 \cdot x \cdot x \cdot x \cdot x$.

In the term $7x^4$, 7 is called the coefficient, x is called the base, and 4 is called the exponent.

Definition: Degree of a Polynomial – is the maximum number of variables that appear as factors in any one term of the polynomial.

Rule: Add the exponents on the variables for each individual term. Whichever term has the highest value, gives the degree of the polynomial.

Example: What is the degree of $7x^2y - 4xy^3z + 6x^2y^2z^3$?

There are 3 terms. The first term is $7x^2y$, and its degree is 3. The second term is $-4xy^3z$, and its degree is 5. The third term is $6x^2y^2z^3$, and its degree is 7. The highest degree value is 7, so that is the degree of the polynomial.

Break for Practice: State the degree of each of the following polynomials.

1. $5x^4 - 3x^3 + 7x + 2$ — 4th degree
2. $4x^3y^2 - 7x^3y^3 + 6xy^2$ — 6th degree
   5 6 3
3. $-2x^2y^3z^4$ — 9th degree
4. $4x^3 - 3x^2y - 7x + 2$ — 3rd degree
   3 3 1 0
5. $5x^2y^3 - 2xy^5 - 7$ — 6th degree
   5 6 0

Definition: <u>Like Terms</u> – like terms have identical variable parts.

Example: $7x^2y$ and $-3x^2y$ are like terms because the variable portion of each term matches. Like terms can be added or subtracted.

Simplifying Polynomials means that all like terms are combined, and we usually arrange the terms in order of decreasing degrees of one of the variables.

Example: Simplify $7xy^3 + 3x^2y - 2x^3 + 6xy^3 - x^2y - 3x^3$

$13xy^3 + 2x^2y - 5x^3$ or $-5x^3 + 2x^2y + 13xy^3$

(decreasing y) (decreasing x)

Break for Practice: Simplify each polynomial.

1. $x^4 - x^3 + 3x^4 - 2x^3 + 3x^2$

   $4x^4 - 3x^3 + 3x^2$

2. $4x^3y^2 + 2x^2y - 6x^3y^2 - 4xy + 7xy + 5x^2y$

   $-2x^3y^2 + 7x^2y + 3xy$

3. $x^3 + 4x^2 - 3x^3 + x - 7x^2 + 8$

$-2x^3 - 3x^2 + x + 8$

4. $2xy - 3yz + 4xz - 2xy + 3yz - 4xz$

0

Polynomials can be added and subtracted by combining like terms. In the case of subtraction, it helps to first distribute a negative one.

Addition Example: $(x^2 - 2x + 1) + (3x^2 + 5x + 4)$

$4x^2 + 3x + 5$

Subtraction Example: $(x^2 - 2x + 1) - (3x^2 + 5x + 4)$

$x^2 - 2x + 1 - 3x^2 - 5x - 4$

$-2x^2 - 7x - 3$

Extended Practice: Simplify each of the following by performing the indicated addition or subtraction.

1. $(3x^2 - 4x + 5) + (2x^2 - 3x - 1)$

$5x^2 - 7x + 4$

2. $(3x^2 - 4x + 5) - (2x^2 - 3x - 1)$

$x^2 - x + 6$

3. $(x^3 - 2x^2) + (3x^2 + 5x + 1)$

$x^3 + x^2 + 5x + 1$

4. $(x^3 - 2x^2) - (3x^2 + 5x + 1)$

$x^3 - 5x^2 - 5x - 1$

5. $(-5x^2 - 2x + 7) + (-2x^2 - 3x + 4)$

$-7x^2 - 5x + 11$

6. $(-5x^2 - 2x + 7) - (-2x^2 - 3x + 4)$

$-3x^2 + x + 3$

7. $3(x^2 - 3x + 4) + 2(3x^2 - 4x - 1)$

$3x^2 - 9x + 12 + 6x^2 - 8x - 2$ $\quad$ $9x^2 - 17x + 10$

8. $3(x^2 - 3x + 4) - 2(3x^2 - 4x - 1)$

$3x^2 - 9x + 12 - 6x^2 + 8x + 2$

$-3x^2 - x + 14$

Using Laws of Exponents

The next skill that needs to be learned when dealing with polynomials is how to simplify expressions by applying the laws of exponents. The exponent laws that we will use in this section should be familiar to you from Algebra I, but it is wise to review.

Example	Law of Exponents
$x^2 \cdot x^3 = x \cdot x\,(x \cdot x \cdot x) = x \cdot x \cdot x \cdot x \cdot x = x^5$	$a^m \cdot a^n = a^{m+n}$
$(x^2)^3 = x^2 \cdot x^2 \cdot x^2 = x^{2+2+2} = x^6$	$(a^m)^n = a^{mn}$
$(x \cdot y)^3 = xy \cdot xy \cdot xy = x \cdot x \cdot x \cdot y \cdot y \cdot y = x^3y^3$	$(ab)^m = a^m b^m$

$x^a + x^a = 2x^a$ (like terms)

Reminder: $x = x^1$

Note: It is important to understand the difference between these two similar looking expressions. Simplify each.

$-4^2 = -16$ $\quad$ $(-4)^2 = 16$

$-(4 \cdot 4)$ $\quad$ $(-4)(-4)$

If you want to square a negative number on your calculator, you must put that negative number in parentheses.

Break for Practice: Simplify

1. $c^4 \cdot c^2$

   c^6

2. $(mn^2)^4$

   m^4n^8

3. $(-x^5)^2$

   x^{10}

4. $5x^3 \cdot 2x^2$

   $10x^5$

5. $(x^2y^3)^5$

   $x^{10}y^{15}$

6. $(6m^4n^3)(2mn)$

   $12m^5n^4$

7. $(6c^2d^4e^5)^2$

$36c^4d^8e^{10}$

8. $(2r^2)^3(3r)^2$

$2^3r^6 \cdot 3^2r^2$

$8r^6 \cdot 9r^2 = 72r^8$

9. $3x^2(5x^2 + 3x - 2)$

$15x^4 + 9x^3 - 6x^2$

10. $a^2b^3(3a^3 - 4a^2b - 2ab^2 + 2)$

$3a^5b^3 - 4a^4b^4 - 2a^3b^5 + 2a^2b^3$

Extended Practice: Simplify

1. $5r^2 \cdot r^4$ $5r^6$	2. $(-t^3)^4$ t^{12}
3. $(4p^2q)(p^2q^3)$ $4p^4q^4$	4. $(r^2s)(-3rs^3)(2rs)$ $-6r^4s^5$
5. $(2c^2d^3)^3$ $8c^6d^9$	6. $(-x^2yz^3)^4$ $x^8y^4z^{12}$
7. $(-c)^2(-c^4)$ $c^2 \cdot (-c^4) = -c^6$	8. $(2x^2y^3)^3(3x^3y)^2$ $8x^6y^9 \cdot 9x^6y^2 = 72x^{12}y^{11}$
9. $x^2(x - 2x^2 + 3x^3)$ $x^3 - 2x^4 + 3x^5$	10. $p^2q^3(p^2 - 4q)$ $p^4q^3 - 4p^2q^4$
11. $t^4 \cdot t^{k-4}$ $t^{4+k-4} = t^k$	12. $y^{p+2} \cdot y^p \cdot y^{p-2}$ y^{3p}
13. $x^3(x^{2k-1})^3$ $x^3(x^{3(2k-1)})$ $x^3 \cdot x^{6k-3} = x^{6k}$	14. $rs^2(r^2 - 2rs - s^2)$ $r^3s^2 - 2r^2s^3 - rs^4$

Multiplying Polynomials

Now that we have reviewed several laws of exponents and used them to multiply monomials, we will extend the ideas to multiplying polynomials.

Do you remember how to multiply and simplify something like this?

$(2x+4)(3x-7)$ F.O.I.L

$6x^2-14x+12x-28 = 6x^2-2x-28$

Break for Practice: Simplify

1. $(a+2)(3a-5)$ $3a^2-5a+6a-10$ $3a^2+a-10$	2. $(7y+z)(2y+5z)$ $14y^2+35yz+2yz+5z^2$ $14y^2+37yz+5z^2$
3. $(y+6)^2 = (y+6)(y+6)$ $y^2+6y+6y+36$ $y^2+12y+36$	4. $(2a-3)^2$ $(2a-3)(2a-3)$ $4a^2-6a-6a+9$ $4a^2-12a+9$

For multiplying larger polynomials, there are a couple of different techniques that can be used.

Example: Multiply $(2x-1)(x^2-3x+5)$ treat like reg. mult.

Horizontal Method

$(2x-1)(x^2-3x+5)$

$2x^3-6x^2+10x-x^2+3x-5$

$2x^3-7x^2+13x-5$

Vertical Method

$$\begin{array}{rrrr} & x^2 & -3x & +5 \\ * & & 2x & -1 \\ \hline & -x^2 & +3x & -5 \\ +\ 2x^3 & -6x^2 & +10x & \\ \hline 2x^3 & -7x^2 & +13x & -5 \end{array}$$

Break for Practice: Simplify

$(x^2+3)(x^4+2x^2-1)$

$x^6 + 2x^4 - x^2 + 3x^4 + 6x^2 - 3$

$x^6 + 5x^4 + 5x^2 - 3$

Extended Practice: Simplify

1. $(4z+3)(3z-4)$ $12z^2 - 16z + 9z - 12$ $12z - 7z - 12$	2. $(4k-5)^2$ $(4k-5)(4k-5)$ $16k^2 - 40k + 25$
3. $(7t+2)(2t-1)$ $14t^2 - 7t + 4t - 2$ $14t^2 - 3t - 2$	4. $(9-5t)(5t-9)$ $45t - 81 - 25t^2 + 45t$ $-25t^2 + 90t - 81$
5. $(2p+3q)(3p-2q)$ $6p^2 - 4pq + 9pq - 6q^2$ $6p^2 + 5pq - 6q^2$	6. $(p^2-2q^2)(p^2+2q^2)$ $p^4 + 2p^2q^2 - 2p^2q^2 - 4q^4$ $p^4 - 4q^4$
7. $(2x^2-5)^2$ $(2x^2-5)(2x^2-5)$ $4x^4 - 10x^2 - 10x^2 + 25$ $\boxed{4x^4 - 20x^2 + 25}$	8. $t(t-2)(t+1)$ $(t^2-2t)(t+1)$ $t^3 + t^2 - 2t^2 - 2t$ $\boxed{t^3 - t^2 - 2t}$
9. $mn(m-n)(m-2n)$ $(m^2n - mn^2)(m-2n)$ $m^3n - 2m^2n^2 - m^2n^2 + 2mn^3$ $\boxed{m^3n - 3m^2n^2 + 2mn^3}$	10. $(x+2)(x^2+3x-5)$ $x^3 + 3x^2 - 5x + 2x^2 + 6x - 10$ $x^3 + 5x^2 + x - 10$
11. $(3-k^2)(2-k^2-k^4)$ $6 - 3k^2 - 3k^4 - 2k^2 + k^4 + k^6$ $k^6 - 2k^4 - 5k^2 + 6$	12. $(a+2b)(a^3-2a^2b-b^3)$ $a^4 - \cancel{2a^3b} - ab^3 + \cancel{2a^3b} - 4a^2b^2 - 2b^4$ $\boxed{a^4 - 4a^2b^2 - ab^3 - 2b^4}$