Algebra II
Unit 5
Rational Expressions

Unit "I can" statements:

1. I can simplify quotients using the laws of exponents.
2. I can simplify expressions involving exponents of zero and negative integers.
3. I can correctly use scientific notation and significant digits.
4. I can simplify rational algebraic expressions.
5. I can graph rational functions.
6. I can multiply and divide rational expressions.
7. I can add and subtract rational expressions.
8. I can solve equations and inequalities that have fractional coefficients.
9. I can solve fractional equations.

Common Core State Standards that are addressed in this unit include: A.SSE.1a, A.SSE.1b, A.CED.1a, A.REI.2a, A.APR.3, A.APR.7d
For more information see www.corestandards.org/Math/

Quotients of Monomials

In this unit we will explore rational algebraic expressions. Since rational numbers are those that can be written as fractions, then rational algebraic expressions must involve both fractions and algebraic expressions. In this unit we will extend many of the same skills that you learned with plain fractions to rational algebraic expressions. We will begin with simply reducing fractions.

Review: Reduce each fraction.

1. $\frac{16 \div 8}{56 \div 8} = \frac{2}{7}$ 2. $\frac{75 \div 25}{125 \div 25} = \frac{3}{5}$ 3. $\frac{63 \div 9}{81 \div 9} = \frac{7}{9}$

In order to reduce rational algebraic expressions, we need two more laws of exponents.

Example	Law of Exponents	
$\left(\frac{2}{5}\right)^3 = \frac{2}{5} \cdot \frac{2}{5} \cdot \frac{2}{5} = \frac{2^3}{5^3} = \frac{8}{125}$	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$	
$\frac{x^5}{x^3} = \frac{\cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot x \cdot x}{\cancel{x} \cdot \cancel{x} \cdot \cancel{x}} = x^2$	$\frac{a^m}{a^n} = a^{m-n}$	*If* $m > n$
$\frac{x^3}{x^5} = \frac{\cancel{x} \cdot \cancel{x} \cdot \cancel{x}}{\cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot x \cdot x} = \frac{1}{x^2}$	$\frac{a^m}{a^n} = \frac{1}{a^{n-m}}$	*If* $n > m$

Now we will use these laws to reduce fractions involving algebraic expressions.

Note: A quotient is reduced when: 1. All common factors are taken out.
2. Each base appears only once.
3. All parentheses are gone.

Break for Practice: Simplify

1. $\frac{18y}{4y^3} = \frac{9}{2y^2}$	2. $\frac{15p^3q^2}{18p^2q^5} = \frac{5p}{6q^3}$

3. $\frac{-8s^4t}{16st^4} = \frac{s^3}{-2t^3}$	4. $\frac{6x^2}{y^2}\cdot\frac{x}{y^2} = \frac{6x^3}{y^4}$
5. $\frac{\cancel{5}x\cancel{y}}{\cancel{2}}\cdot\frac{\cancel{8}x^2}{\cancel{15}\cancel{y^3}} = \frac{4x^3}{3y^2}$	6. $\frac{p^3}{q}\left(\frac{3q}{p}\right)^2 = \frac{p^3}{q}\cdot\frac{9q^2}{p^2} = 9pq$
7. $\frac{(x^2y^3)^2}{(x^3y)^2} = \frac{x^4y^6}{x^6y^2} = \frac{y^4}{x^2}$	8. $\frac{(ab^2c)^2}{(a^3bc^2)^3} = \frac{a^2b^4c^2}{a^9b^3c^6} = \frac{b}{a^7c^4}$

Extended Practice: Simplify

1. $\frac{-12p^3q}{4p^3q^2} = \frac{-3}{q}$	2. $\frac{30x^2y^3}{-6x^3y^2} = \frac{-5y}{x}$
3. $\left(\frac{3r}{s^2}\right)^3$ $\frac{27r^3}{s^6}$	4. $\left(\frac{2x^2}{-y}\right)^4 = \frac{16x^8}{y^4}$
5. $\frac{3x^{\cancel{2}}}{\cancel{y^2}}\cdot\frac{\cancel{3y}}{\cancel{6x}} = \frac{3x}{2y}$	6. $\frac{x\cancel{y^2}}{\cancel{2}}\cdot\frac{\cancel{6}x}{\cancel{y^2}} = 3x^2$
7. $\frac{(xyz^2)^2}{(x^2yz)^2} = \frac{x^2y^2z^4}{x^4y^2z^2} = \frac{z^2}{x^2}$	8. $\frac{(pq^2r^3)^3}{(p^3qr^2)^2} = \frac{p^3q^6r^9}{p^6q^2r^4} = \frac{q^4r^5}{p^3}$

9. $\frac{u^2}{v}\left(\frac{3v}{u^2}\right)^2 = \frac{\cancel{u^2}}{\cancel{v}} \cdot \frac{9v^{\cancel{2}}}{u^{4\,2}} = \frac{9v}{u^2}$	10. $\left(\frac{2x^2}{y^3}\right)^3\left(\frac{-y^3}{2x^2}\right)^2 = \frac{\cancel{8}^{2}x^{\cancel{6}\,2}}{\cancel{y^9}^{3}} \cdot \frac{\cancel{y^6}}{\cancel{4}\cancel{x^4}}$ $\frac{2x^2}{y^3}$

Zero and Negative Exponents

In this section we will see how two new properties of exponents can help us simplify rational algebraic expressions.

Property	Explanation
$a^0 = 1$	Consider: $\frac{x^2}{x^2} = 1$ $\frac{x^2}{x^2} = x^0$ so $x^0 = 1$
$a^{-n} = \frac{1}{a^n}$ Note: $\frac{1}{a^{-n}} = a^n$	consider: $\frac{x^3}{x^5} = \frac{\cancel{x}\cancel{x}\cancel{x}}{\cancel{x}\cancel{x}\cancel{x}xx} = \frac{1}{x^2}$ so $x^{-2} = \frac{1}{x^2}$ $\frac{x^3}{x^5} = x^{3-5} = x^{-2}$

Break for Practice: Write in simplest form without negative or zero exponents.

1. $4 \cdot 3^{-2} = 4 \cdot \frac{1}{3^2} = \frac{4}{9}$	2. $(4^{-1} \cdot 6^{-1} \cdot 7^0)^{-1} = 4 \cdot 6 \cdot 7^0$ $24 \cdot 1 = 24$
3. $\left(\frac{5}{2}\right)^{-2} = \left(\frac{2}{5}\right)^2 = \frac{4}{25}$	4. $\frac{s^{-3}t^{-4}}{s^{-2}t^0} = \frac{s^2}{s^3t^4 \cdot 1} = \frac{1}{st^4}$
5. $\left(\frac{2}{x^2y^{-3}}\right)^{-2} = \frac{2^{-2}}{x^{-4}y^6} = \boxed{\frac{x^4}{4y^6}}$	6. $\frac{9ab^{-2}}{-3a^{-3}b^{-1}} = \frac{-3aba^3}{b^2} = \frac{-3a^4}{b}$
7. $\frac{(2a^{-2}b)^{-3}}{ab^4}\left(\frac{a^{-2}}{b^{-3}}\right)^{-1} = \frac{2^{-3}a^6b^{-3}}{\cancel{a}b^4} \cdot \frac{a^{\cancel{2}}}{b^3} = \frac{a^6 \cdot a}{2^3 \cdot b^3b^4b^3} = \frac{a^7}{8 \cdot b^{10}}$	

Sometimes expressions are considered simplified if no fractions are used.

Break for Practice: Write without using fractions.

1. $\frac{a^3}{b^2}$ a^3b^{-2}	2. $\frac{6x^3}{yz^2}$ $6x^3y^{-1}z^{-2}$

Extended Practice: Write in simplest form without zero or negative exponents.

1. $3 \cdot 5^{-1}$ $\frac{3}{5}$	2. $(3 \cdot 5)^{-1}$ $15^{-1} = \frac{1}{15}$
3. $(-3^{-1})^{-2}$ $(-1)^{-2}(3^2) = \frac{1}{1^2} \cdot 9 = 9$	4. $2\left(\frac{2}{5}\right)^{-2}$ $2\left(\frac{2^{-2}}{5^{-2}}\right) = \frac{2 \cdot 5^2}{2^2} = \frac{50}{4} = \boxed{\frac{25}{2}}$
5. $\frac{p^{-1}q^{-2}}{p^{-3}} = \frac{p^3q^{-2}}{p} = \frac{p^2}{q^2}$	6. $\frac{s^{-2}t^{-3}}{s^{-1}t^0} = \frac{s}{s^2t^3t^0} = \boxed{\frac{1}{st^3}}$
7. $\frac{6xy^{-1}}{-2x^{-2}y^{-1}}$ $\frac{-3xx^2y}{y} = -3x^3$	8. $\left(\frac{u^{-2}}{v}\right)^{-1}$ $\frac{u^2}{v^{-1}} = u^2v^1$
9. $\left(\frac{2}{h^2k^{-3}}\right)^{-2} = \frac{2^{-2}}{h^{-4}k^6} = \frac{h^4}{4k^6}$	10. $\frac{(3x^{-2}y)^{-1}}{(2xy^{-2})^0} = \frac{3^{-1}x^2y^{-1}}{1} = \frac{x^2}{3y}$
11. $\left(\frac{a^0}{b}\right)^{-2}\left(\frac{a}{b^{-2}}\right)^{-2}(ab^2)^{-1}$ $\frac{1}{b^{-2}} \cdot \frac{a^{-2}}{b^4} \cdot \frac{1}{ab^2} = \frac{b^2}{a^2b^4ab^2} = \frac{1}{a^3b^4}$	
12. $\left(\frac{17x^{-2}y^{-3}z^4}{34x^{-8}y^7z^{-3}}\right)^0 = 1$	

Write without using fractions.

1. $\frac{6x^3}{y^3}$ $6x^3y^{-3}$	2. $\frac{x^3}{yz^4}$ $x^3y^{-1}y^{-4}$

Scientific Notation and Significant Digits

One common place where negative exponents are found is in scientific notation. What do you already know about scientific notation?

Definition: A number is in **scientific notation** when it is in the form $m \times 10^n$ $1 \leq m < 10$

"m" is called the mantissa $n \in$ integers

"n" is called the characteristic

Break for Practice: Complete the chart.

Rewrite in decimal form	Rewrite in Scientific Notation
3.17×10^5 = 317,000	32,000 3.2×10^4
5.143×10^{-2} = 0.05143	0.0001084 1.084×10^{-4}
-2.5×10^3 = −2,500	39648.1 3.96481×10^4
-3.4×10^{-1} = −.34	0.1048 1.048×10^{-1}
10^3 = 1000	500 5×10^2
10^{-4} = 0.001	4,151,964,000,000 4.151964×10^{12}

In many applications, it is important to be aware of significant digits. These are very important in the sciences. When a number is in scientific notation, the number of digits in the mantissa is the number of significant digits. When a number is in decimal form, it is a little more complicated.