Extended Practice: Simplify

1. $\frac{5x^2-15x}{10x^2} = \frac{\cancel{5}\cancel{x}(x-3)}{2\,\cancel{10}x^{\cancel{2}1}} = \boxed{\frac{(x-3)}{2x}}$

2. $\frac{3t^4-9t^3}{6t^2} = \frac{3t^3(t-3)}{6t^2} = \frac{t(t-3)}{2}$

3. $\frac{u^2-u-2}{u^2+u} = \frac{(u-2)\cancel{(u+1)}}{u\cancel{(u+1)}} = \boxed{\frac{(u-2)}{u}}$

4. $\frac{z^3-4z}{z^2-4z+4} = \frac{z(z^2-4)}{(z-2)(z-2)} = \frac{z\cancel{(z-2)}(z+2)}{\cancel{(z-2)}(z-2)} = \boxed{\frac{z(z+2)}{(z-2)}}$

5. $(p-q)(q-p)^{-1} = \frac{(p-q)}{(q-p)} = \frac{\cancel{(p-q)}}{-1\cancel{(p-q)}} = \frac{1}{-1} = -1$

6. $(r^2 - rs)(r^2 - s^2)^{-1}$

$$\frac{(r^2 - rs)}{(r^2 - s^2)} = \frac{r(\cancel{r-s})}{(\cancel{r-s})(r+s)} = \boxed{\frac{r}{(r+s)}}$$

7. $\frac{s^2-t^2}{(t-s)^2} = \frac{(s-t)(s+t)}{(t-s)(t-s)} = \frac{-1(\cancel{t-s})(s+t)}{(\cancel{t-s})(t-s)} = \boxed{\frac{-(s+t)}{(t-s)}}$

8. $\frac{(a-x)^2}{x^2-a^2} = \frac{(a-x)(a-x)}{(x-a)(x+a)} = \frac{-1(\cancel{x-a})(a-x)}{(\cancel{x-a})(x+a)} = \boxed{\frac{-(a-x)}{(x+a)}}$

9. $\frac{x^2-5x+6}{x^2-7x+12} = \frac{(x-2)(\cancel{x-3})}{(\cancel{x-3})(x-4)} = \boxed{\frac{(x-2)}{(x-4)}}$

10. $\frac{2t^2+5t-3}{2t^2+7t+3} = \frac{(2t-1)(\cancel{t+3})}{(2t+1)(\cancel{t+3})} = \boxed{\frac{(2t-1)}{(2t+1)}}$

Graphing Rational Functions

This section will investigate the graphs of rational functions. Rational functions can include unique features such
as holes and asymptotes. Since graphing calculators do not always show these features well, it is important to know how to algebraically find these features.

Example: Graph the following function and identify the locations of all holes and/or vertical asymptotes.

$$f(x) = \frac{x^2 - 1}{x^2 + 2x - 3} \overset{1}{=} \frac{\cancel{(x-1)}(x+1)}{(x+3)\cancel{(x-1)}}$$

2) $x+3=0$
$-3 \quad -3$
$x=-3$

$x-1=0$
$+1 \quad +1$
$x=1$

$\boxed{x \neq -3, 1}$ excluded values

$f(x) \overset{3}{=} \boxed{\frac{(x+1)}{(x+3)}}$ reduced fraction

$x=-3: \frac{-3+1}{-3+3} = \frac{-2}{0}$

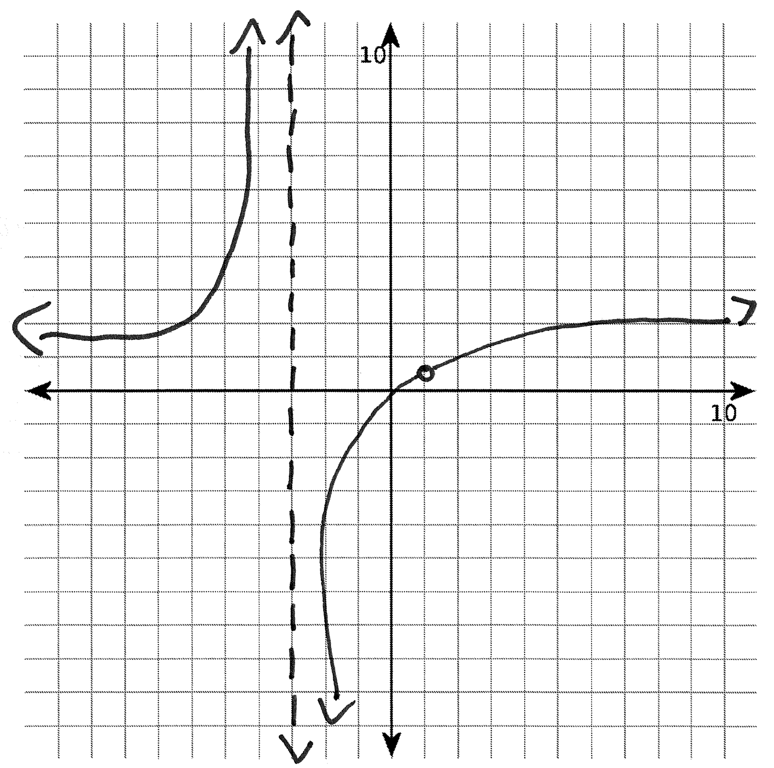
$\boxed{\text{vert. asy. } x=-3}$

$x=1: \frac{1+1}{1+3} = \frac{2}{4} = \frac{1}{2}$

$\boxed{\text{hole @ } (1, \tfrac{1}{2})}$

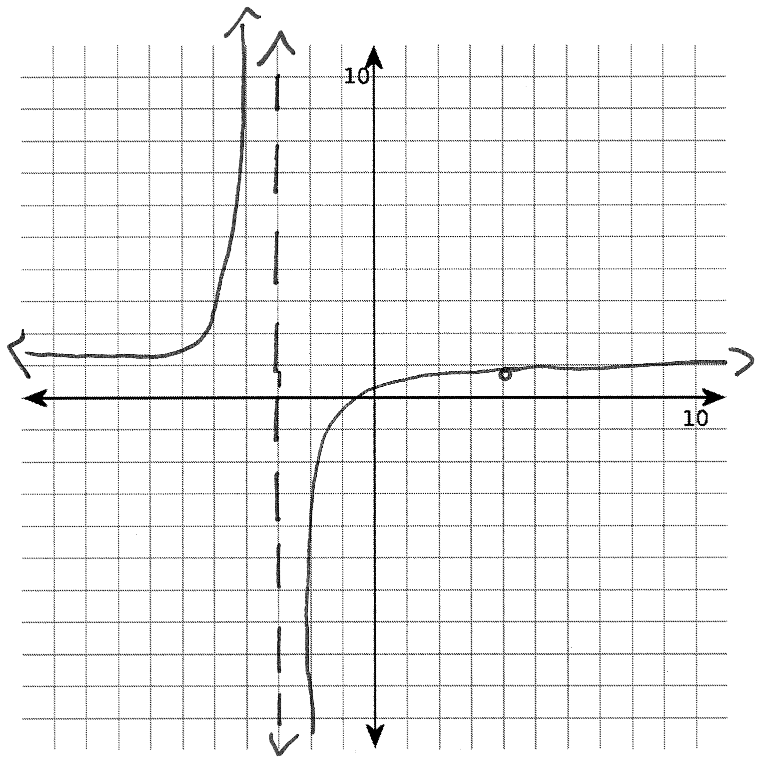
Steps:

1. Simplify the function by factoring.
2. Identify excluded values. (These are the values that make the denominator equal zero.)
3. Find the vertical asymptotes and/or holes by testing the excluded values in the reduced function.
   a) If you get $\frac{\#}{0}$, then it's a vertical asymptote.
   b) If you get $\frac{\#}{\#}$, then it's a hole.
4. Complete the graph with the graphing calculator. (Use orig. function with parentheses



Break for Practice: Graph the following function and identify the locations of all holes and/or vertical asymptotes.

1. $f(x) = \frac{x^2-2x-8}{x^2-x-12} = \frac{\cancel{(x-4)}(x+2)}{\cancel{(x-4)}(x+3)}$



excluded values

$x-4=0 \quad x+3=0$

$+4 \quad +4 \quad\quad -3 \quad -3$

$\boxed{x \neq 4, -3}$

$f(x) = \frac{x+2}{x+3}$

$x=4: f(4) = \frac{4+2}{4+3}$

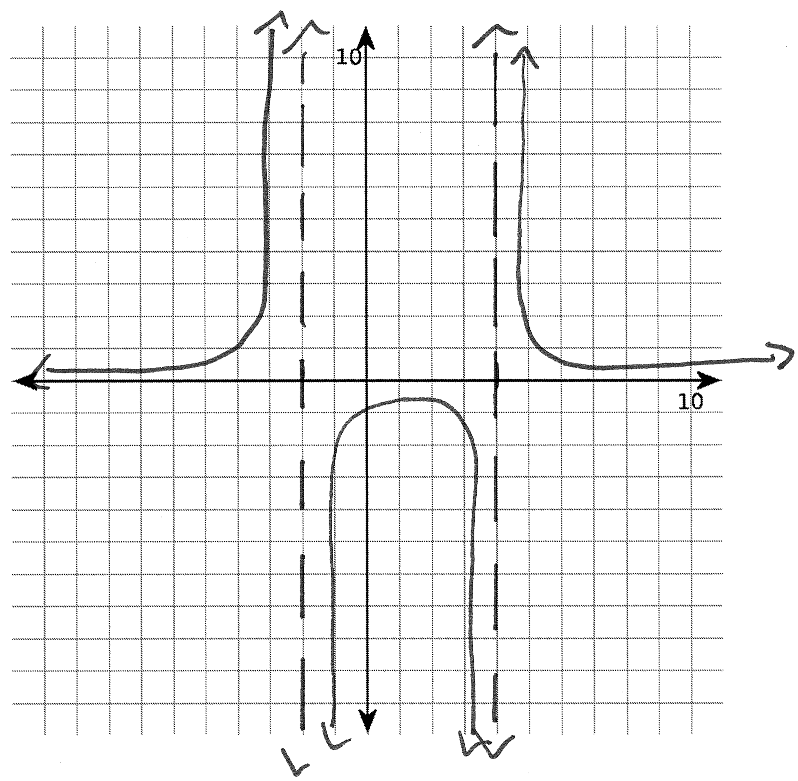
$f(4) = \frac{6}{7}$

$\boxed{\text{hole at } (4, \tfrac{6}{7})}$

$f(-3) = \frac{-3+2}{-3+3} = \frac{-1}{0}$

$\boxed{\text{vert asy @ } x=-3}$

2. $f(x) = \frac{5}{x^2-2x-8}$



$f(x) = \frac{5}{(x-4)(x+2)}$

excl. values

$x-4=0 \quad x+2=0$

$+4 \quad +4 \quad\quad -2 \quad -2$

$\boxed{x \neq 4, -2}$

$f(4) = \frac{5}{(4-4)(4+2)} = \frac{5}{0}$ vert. asy @ $x=4$

$f(-2) = \frac{5}{(-2-4)(-2+2)} = \frac{5}{0}$ vert. asy @ $x=-2$

3. $f(x) = \frac{x^2+6x-7}{x-1} = \frac{(x+7)\cancel{(x-1)}}{\cancel{(x-1)}}$

excluded value

$x-1=0$
$+1 \quad +1$

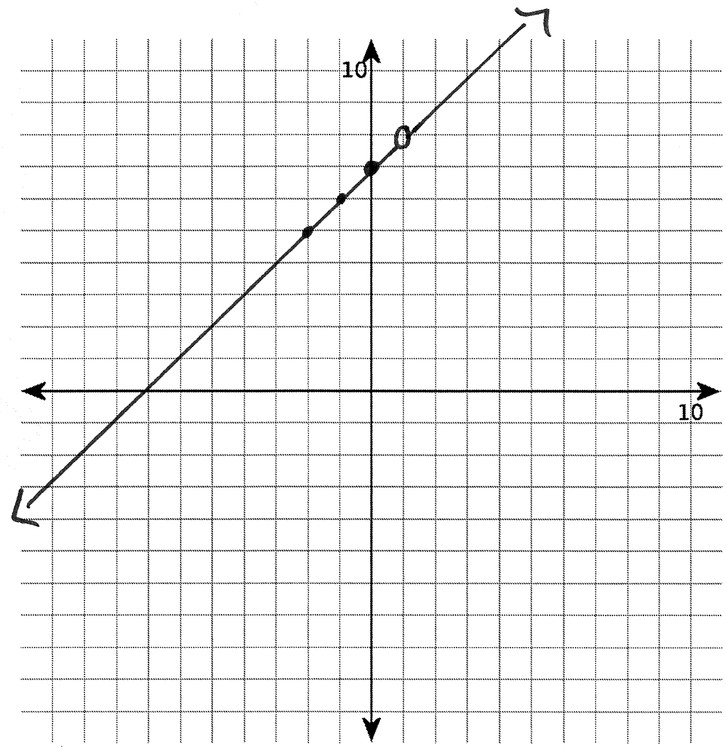
$\boxed{x \neq 1}$

$f(x) = x+7$

$f(1) = 1+7$

$f(1) = 8$

$\boxed{\text{hole @ } (1,8)}$



Extended Practice: Graph the following function and identify the locations of all holes and/or vertical asymptotes.

1. $f(x) = \frac{x^2-4x+3}{x^2-x-6} = \frac{\cancel{(x-3)}(x-1)}{\cancel{(x-3)}(x+2)}$

excl. values

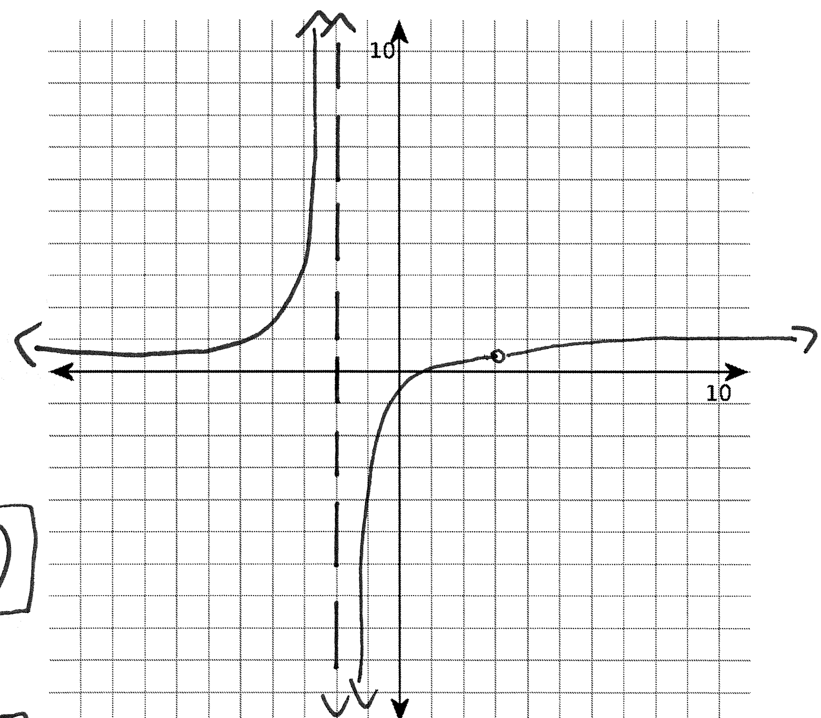
$x-3=0 \qquad x+2=0$
$+3 \quad +3 \qquad -2 \quad -2$

$\boxed{x \neq 3, -2}$

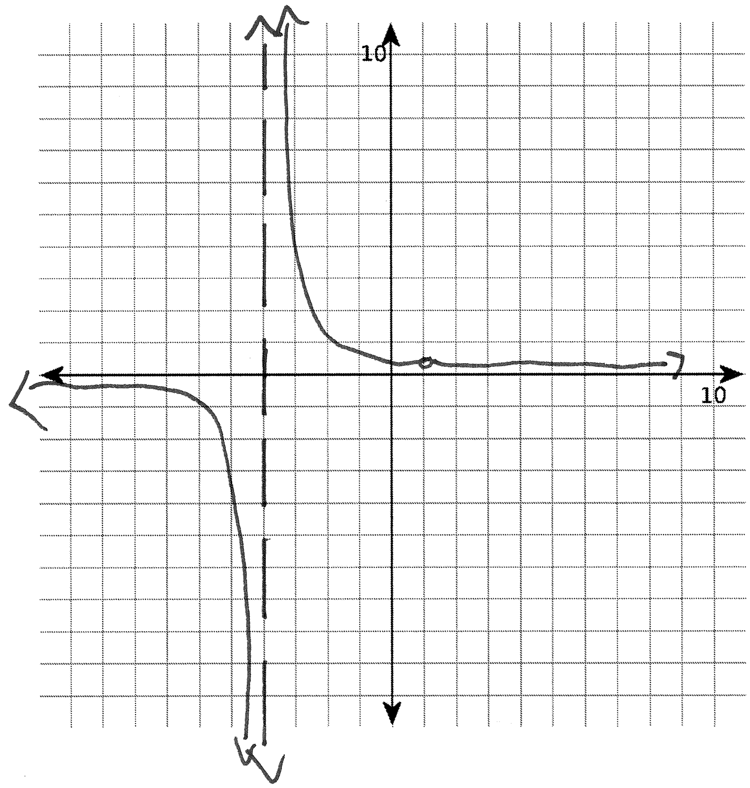
$f(x) = \frac{(x-1)}{(x+2)}$

$f(3) = \frac{3-1}{3+2} = \frac{2}{5}$ $\boxed{\text{hole @ } (3, \tfrac{2}{5})}$

$f(-2) = \frac{-2-1}{-2+2} = \frac{-3}{0}$ $\boxed{\text{vert asy. @ } x=-2}$



2. $f(x) = \frac{x-1}{x^2+3x-4} = \frac{(\cancel{x-1})}{(x+4)(\cancel{x-1})}$



excl. values

$x+4=0 \quad x-1=0$

$-4 \;\; -4 \quad +1 \;\; +1$

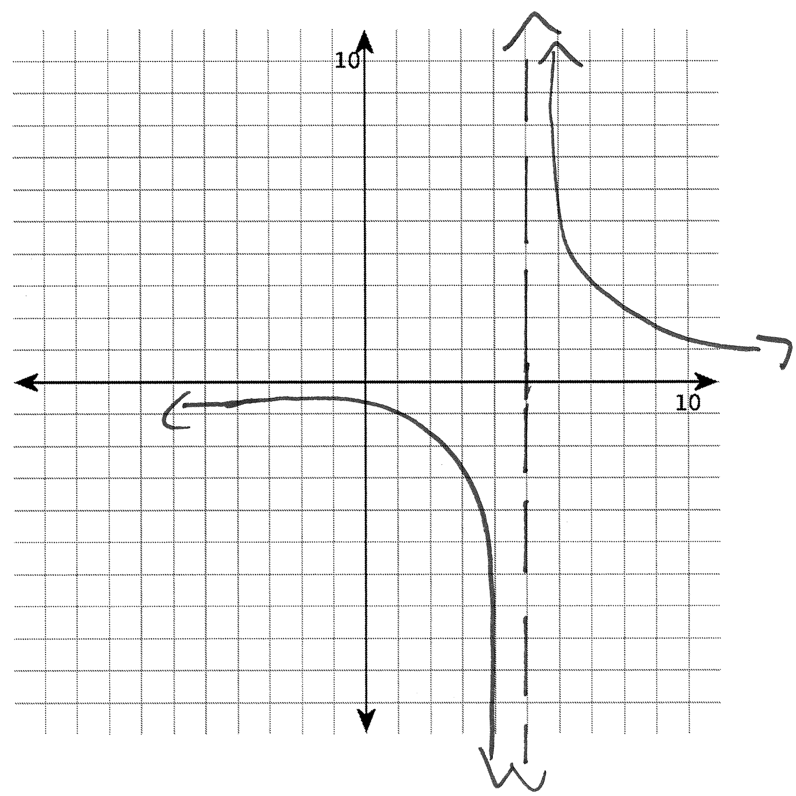
$x=-4 \quad x=1 \quad \boxed{x \neq -4, 1}$

$f(x) = \frac{1}{(x+4)}$

$f(-4) = \frac{1}{-4+4} = \frac{1}{0}$ $\boxed{\text{vert asy @ } x=-4}$

$f(1) = \frac{1}{1+4} = \frac{1}{5}$ $\boxed{\text{hole @} (1, \tfrac{1}{5})}$

3. $f(x) = \frac{4}{x-5}$ *reduced already



excl. values

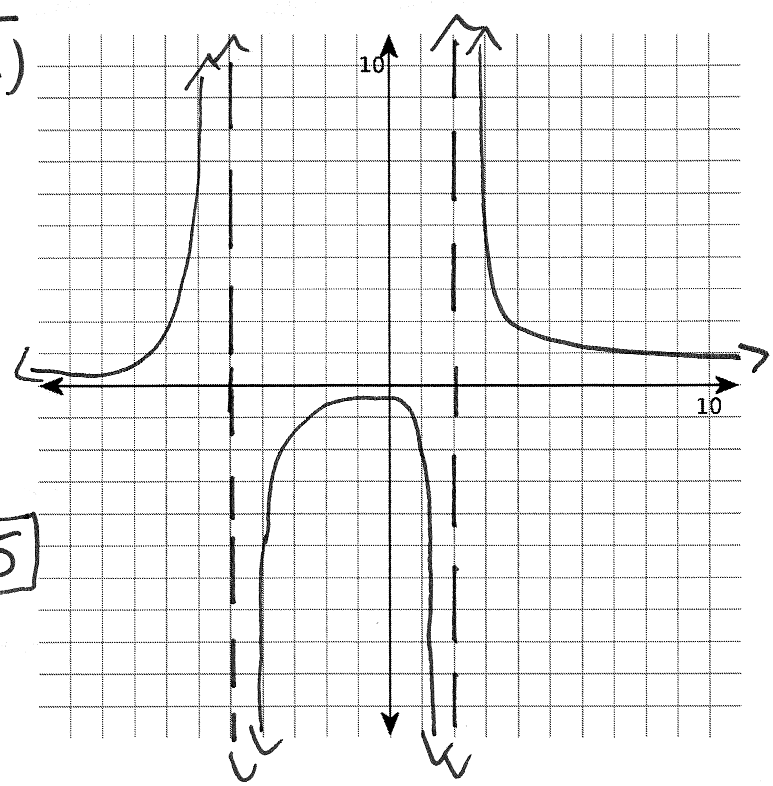
$x-5=0$

$+5 \;\; +5$

$\boxed{x \neq 5}$

$f(5) = \frac{4}{5-5} = \frac{4}{0}$ vert asy. @ $x=5$

4. $f(x) = \dfrac{2}{x^2 + 3x - 10} = \dfrac{2}{(x+5)(x-2)}$



excl. values

$x+5=0 \qquad x-2=0$

$-5 \quad -5 \qquad +2 \quad +2$

$\boxed{x \neq -5, 2}$

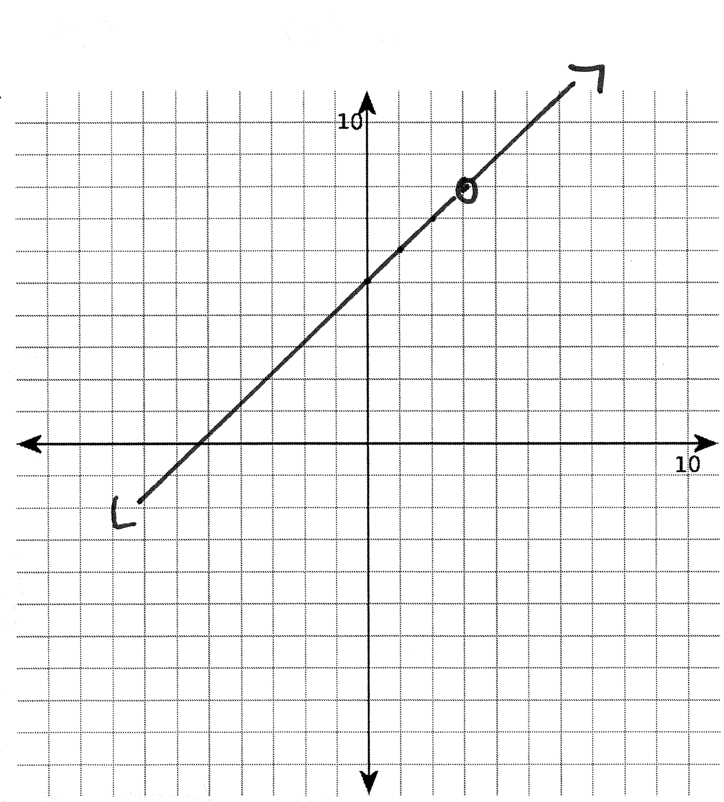
$f(-5) = \dfrac{2}{(-5+5)(-5-2)} = \dfrac{2}{0}$

$\boxed{\text{vert. asy @ } x=-5}$

$f(2) = \dfrac{2}{(2+5)(2-2)} = \dfrac{2}{0}$

$\boxed{\text{vert asy @ } x=2}$

5. $f(x) = \dfrac{x^2 + 2x - 15}{x - 3} = \dfrac{(x+5)\cancel{(x-3)}}{\cancel{(x-3)}}$



excl. values

$x-3=0$

$+3 \quad +3$

$\boxed{x \neq 3}$

$f(x) = x+5$

$f(3) = 3+5$

$f(3) = 8$

$\boxed{\text{hole @ } (3,8)}$

Products and Quotients of Rational Algebraic Expressions

In this section we will learn how to multiply and divide rational expressions. We will begin by reviewing the techniques with simple numerical fractions.

Review: Simplify

1. $\frac{\cancel{15}^{3}}{\cancel{24}_{4}} \cdot \frac{\cancel{18}^{3}}{\cancel{25}_{5}} = \frac{9}{20}$

2. $\frac{12}{32} \div \frac{18}{24}$ $\quad \frac{\cancel{12}^{\cancel{2}\,1}}{\cancel{32}_{\cancel{4}\,2}} \cdot \frac{\cancel{24}^{\cancel{3}\,1}}{\cancel{18}_{\cancel{3}\,1}} = \boxed{\frac{1}{2}}$

Now it is time to try the same thing with rational algebraic expressions. The same technique applies, but polynomials should be factored before any reducing takes place.

Break for Practice: Simplify

1. $\frac{1}{\cancel{x+2}} \cdot \frac{x^2-4}{1}$ $\quad (x-2)\cancel{(x+2)}$ $\quad = \frac{x-2}{1} = \boxed{x-2}$

2. $\frac{x+3}{x-5} \div \frac{1}{x^2-2x-15}$ $\quad (x-5)(x+3)$ $\quad = \frac{(x+3)}{\cancel{(x-5)}} \cdot \frac{\cancel{(x-5)}(x+3)}{1} = (x+3)(x+3)$

$\boxed{(x+3)^2}$

3. $\frac{x^2+3x-10}{x^2-7x+6} \cdot \frac{x^2+2x-3}{x^2+x-6} = \frac{(x+5)\cancel{(x-2)}}{(x-6)\cancel{(x-1)}} \cdot \frac{\cancel{(x+3)}\cancel{(x-1)}}{\cancel{(x+3)}\cancel{(x-2)}} = \boxed{\frac{(x+5)}{(x-6)}}$

4. $\frac{(x-3)^2}{9+3x} \div \frac{9-x^2}{x^2+3x} = \frac{(x-3)(x-3)}{3(3+x)} \div \frac{(3-x)(3+x)}{x(x+3)}$

$= \frac{(x-3)\cancel{(x-3)}}{3(3+x)} \cdot \frac{x\cancel{(x+3)}}{\cancel{(3-x)}_{-1}\cancel{(3+x)}} = \boxed{\frac{x(x-3)}{-3(3+x)}}$