3. $\sqrt[3]{4x}+7=5$

$-7 \quad -7$

$(\sqrt[3]{4x})^3=(-2)^3$

$\frac{4x}{4}=\frac{-8}{4}$

$\boxed{x=-2}$

Check

$\sqrt[3]{4(-2)}+7=5$

$\sqrt[3]{-8}+7=5$

$-2+7=5$ ✓

4. $(\sqrt[3]{x-2})^3=2^3$

$x-2=8$

$+2 \quad +2$

$\boxed{x=10}$

$\sqrt[3]{10-2}=2$

$\sqrt[3]{8}=2$ ✓

5. $3=x+\sqrt{x-1}$

$-x \quad -x$

$(3-x)^2=(\sqrt{x-1})^2$

$(3-x)(3-x)=x-1$

$9-3x-3x+x^2=x-1$

$+1 \quad -x \quad -x+1$

$x^2-7x+10=0$

$(x-5)(x-2)=0$

$x-5=0 \quad x-2=0$

$+5 \quad +5 \quad +2 \quad +2$

$\boxed{x=2,\cancel{5}}$

$3=5+\sqrt{5-1}$

$3=5+\sqrt{4}$

$3\neq 5+2$ NO

$3=2+\sqrt{2-1}$

$3=2+\sqrt{1}$

$3=2+1$ ✓

6. $\sqrt{x+6}-x=4$

$+x \quad +x$

$(\sqrt{x+6})^2=(x+4)^2$

$x+6=(x+4)(x+4)$

$x+6=x^2+4x+4x+16$

$-x-6 \quad -x \quad -6$

$0=x^2+7x+10$

$0=(x+5)(x+2)$

$x+5=0$

$-5 \quad -5$

$x=-5$

$x+2=0$

$-2 \quad -2$

$x=-2$

$\boxed{x=\cancel{-5},-2}$

$\sqrt{-5+6}-(-5)=4$

$\sqrt{1}+5\neq 4$

$\sqrt{-2+6}-(-2)=4$

$\sqrt{4}+2=4$

$2+2=4$ ✓

Extended Practice: Solve each radical equation and identify any extraneous solutions.

1. $(\sqrt{4x-3})^2=5^2$ $4x-3=25$ $+3 \quad +3$ $\frac{4x}{4}=\frac{28}{4}$ $\boxed{x=7}$ $\sqrt{28-3}=5$ $\sqrt{25}=5$ ✓	2. $(\sqrt{3n+1})^2=7^2$ $3n+1=49$ $-1 \quad -1$ $\frac{3n}{3}=\frac{48}{3}$ $\boxed{n=16}$ $\sqrt{3(16)+1}=7$ $\sqrt{49}=7$ ✓
3. $(\sqrt[3]{3m+1})^3=4^3$ $3m+1=64$ $-1 \quad -1$ $\frac{3m}{3}=\frac{63}{3}$ $\boxed{m=21}$ $\sqrt[3]{3(21)+1}=4$ $\sqrt[3]{64}=4$ ✓	4. $7-\sqrt[3]{9c}=4$ $-7 \quad -7$ $\frac{-\sqrt[3]{9c}}{-1}=\frac{-3}{-1}$ $(\sqrt[3]{9c})^3=3^3$ $\frac{9c}{9}=\frac{27}{9}$ $\boxed{c=3}$ $7-\sqrt[3]{9(3)}$ $7-3=4$ ✓
5. $(\sqrt{x+2})^2=x^2$ $x+2=x^2$ $-x-2 \quad -x-2$ $0=x^2-x-2$ $0=(x-2)(x+1)$ $x-2=0 \quad x+1=0$ $+2 \quad +2 \quad -1 \quad -1$ $\boxed{x=2},\cancel{-1}$ $\sqrt{2+2}=2$ $\sqrt{4}=2$ ✓ $\sqrt{-1+2}$ $\sqrt{1}\neq -1$	6. $(\sqrt{2n+3})^2=n^2$ $2n+3=n^2$ $-2n-3 \quad -2n-3$ $0=n^2-2n-3$ $0=(n-3)(n+1)$ $n-3=0 \quad n+1=0$ $+3 \quad +3 \quad -1 \quad -1$ $\boxed{n=3},\cancel{-1}$ $\sqrt{2(3)+3}=3$ $\sqrt{6+3}=3$ ✓ $\sqrt{2(-1)+3}=-1$ $\sqrt{1}\neq -1$
7. $5+\sqrt{a+7}=a$ $-5 \quad -5$ $(\sqrt{a+7})^2=(a-5)^2$ $a+7=(a-5)(a-5)$ $a+7=a^2-5a-5a+25$ $-a-7 \quad -a \quad -7$ $0=a^2-11a+18$ $(a-9)(a-2)$ $0=(a-9)(a-2)$ $a-9=0 \quad a-2=0$ $+9 \quad +9 \quad +2 \quad +2$ $\boxed{a=9}\ \cancel{2}$ $5+\sqrt{9+7}=9$ $5+4=9$ ✓ $5+\sqrt{2+7}=2$ $5+3\neq 2$	8. $\sqrt{2x+5}-1=x$ $+1 \quad +1$ $(\sqrt{2x+5})^2=(x+1)^2$ $2x+5=(x+1)(x+1)$ $2x+5=x^2+x+x+1$ $-2x-5 \quad -2x-5$ $0=x^2-4$ $+4 \quad +4$ $\sqrt{4}=\sqrt{x^2}$ $\pm 2=x$ $\boxed{x=2}$ $\sqrt{2(2)+5}-1=2$ $\sqrt{9}-1=2$ ✓ $\sqrt{2(-2)+5}-1$ $\sqrt{1}-1\neq -2$

9. If you are near the top of a tall building on a clear day, how far can you see? If a building is h feet high, then the distance d (in miles) to the earth's horizon is approximately $d = \sqrt{\frac{3}{2}h}$

a) The observatory of a tall building in Chicago is 607 feet high. What is the distance to the horizon from this observatory?

$d = \sqrt{\frac{3}{2}(607)}$

$d \approx \boxed{30.2 \text{ miles}}$

b) Solve the formula for h.

$(d)^2 = \left(\sqrt{\frac{3}{2}h}\right)^2$

$\frac{2}{3}(d^2) = \left(\frac{3}{2}h\right)\frac{2}{3}$

$\boxed{h = \frac{2}{3}d^2}$

10. If a pendulum is l cm long, then the time T (in seconds) that it takes the pendulum to swing back and forth once is given by $T = 2\pi\sqrt{\frac{l}{g}}$, where $\pi \approx 3.14$ *and* $g \approx 980$.

a) Find the value of T if the pendulum is 20 cm long.

$T = 2\pi\sqrt{\frac{20}{980}} \Rightarrow 2\pi\sqrt{\frac{1}{49}} = 2\pi\left(\frac{1}{7}\right)$

$\boxed{T \approx .9 \text{ sec}}$

b) Solve the formula for l in terms of T, g, and π.

$\frac{T}{2\pi} = \frac{2\pi\sqrt{\frac{l}{g}}}{2\pi}$

$\left(\frac{T}{2\pi}\right)^2 = \left(\sqrt{\frac{l}{g}}\right)^2$

$g\left(\frac{T^2}{4\pi^2}\right) = \left(\frac{l}{g}\right)g$

$\boxed{l = g\left(\frac{T^2}{4\pi^2}\right)}$

All of the previous problems had only one radical. Now we shall see how to use our steps for solving when there are multiple radicals in the equations.

Break for Practice: Solve each radical equation and identify any extraneous solutions.

1. $\sqrt{2x-4} - \sqrt{x-3} = 1$

$+\sqrt{x-3} \quad +\sqrt{x-3}$

$\left(\sqrt{2x-4}\right)^2 = \left(1 + \sqrt{x-3}\right)^2$

$2x - 4 = (1 + \sqrt{x-3})(1 + \sqrt{x-3})$

$2x - 4 = 1 + 1\sqrt{x-3} + 1\sqrt{x-3} + x - 3$

$2x - 4 = x - 2 + 2\sqrt{x-3}$

$-x \ +2 \quad -x \ +2$

$(x-2)^2 = \left(2\sqrt{x-3}\right)^2$

$(x-2)(x-2) = 2^2\left(\sqrt{x-3}\right)^2$

$x^2 - 2x - 2x + 4 = 4(x-3)$

$x^2 - 4x + 4 = 4x - 12$

$-4x + 12 \quad -4x + 12$

$x^2 - 8x + 16 = 0$

$(x-4)(x-4) = 0$

$x - 4 = 0$

$+4 \quad +4$

$\boxed{x = 4}$

$\sqrt{2(4)-4} - \sqrt{4-3} = 1$

$\sqrt{4} - \sqrt{1} = 1$

$2 - 1 = 1$ ✓