2. $2\sqrt{x+4} - \sqrt{2x+25} = 1$

$+\sqrt{2x+25} \quad +\sqrt{2x+25}$

$(2\sqrt{x+4})^2 = (1+\sqrt{2x+25})^2$

$(1+\sqrt{2x+25})(1+\sqrt{2x+25})$

$4(x+4) = 1 + \sqrt{2x+25} + \sqrt{2x+25} + 2x+25$

$4x+16 = 1 + 2\sqrt{2x+25} + 2x + 25$

$-2x - 26 \qquad -2x \quad -26$

$\frac{2x-10}{2} = \frac{2\sqrt{2x+25}}{2}$

$(x-5)^2 = (\sqrt{2x+25})^2$

$(x-5)(x-5)$

$x^2 - 5x - 5x + 25 = 2x + 25$

$x^2 - 10x + 25 = 2x + 25$

$-2x - 25 \quad -2x - 25$

$x^2 - 12x = 0$

$x(x-12) = 0$

$x = \cancel{0} \qquad x - 12 = 0$

$+12 \quad +12$

$\boxed{x = 12}$

$x=0$

$2\sqrt{0+4} - \sqrt{2(0)+25} = 1$

$2\sqrt{4} - \sqrt{25} = 1$

$2 \cdot 2$

$4 - 5 \neq 1$

$x = 12$

$2\sqrt{12+4} - \sqrt{2(12)+25} =$

24

$2\sqrt{16} - \sqrt{49} = 1$

$2 \cdot 4$

$8 - 7 = 1$ ✓

Extended Practice: Solve each radical equation and identify any extraneous solutions.

1. $\sqrt{y} + \sqrt{y+5} = 5$

$-\sqrt{y} \quad -\sqrt{y}$

$(\sqrt{y+5})^2 = (5-\sqrt{y})^2$

$(5-\sqrt{y})(5-\sqrt{y})$

$\cancel{y} + 5 = \cancel{25} - 5\sqrt{y} - 5\sqrt{y} + \cancel{y}$

$-\cancel{y} - 25 \quad -\cancel{25} \quad -\cancel{y}$

$\frac{-20}{-10} = \frac{-10\sqrt{y}}{-10}$

$(2)^2 = (\sqrt{y})^2$

$\boxed{4 = y}$

$\sqrt{4} + \sqrt{4+5} = 5$

$\sqrt{9}$

$2 + 3 = 5$ ✓

2. $\sqrt{x-7}+\sqrt{x}=7$

$-\sqrt{x} \quad -\sqrt{x}$

$(\sqrt{x-7})^2=(7-\sqrt{x})^2$

$(7-\sqrt{x})(7-\sqrt{x})$

$x-7=49-7\sqrt{x}-7\sqrt{x}+x$

$-x-49 \quad -49 \qquad -x$

$\frac{-56}{-14}=\frac{-14\sqrt{x}}{-14}$

$(4)^2=(\sqrt{x})^2$

$\boxed{16=x}$

$\sqrt{16-7}+\sqrt{16}=7$

$\sqrt{9}+4$

$3+4=7$ ✓

3. $\sqrt{3a-2}-\sqrt{2a-3}=1$

$+\sqrt{2a-3} \quad +\sqrt{2a-3}$

$(\sqrt{3a-2})^2=(1+\sqrt{2a-3})^2$

$(1+\sqrt{2a-3})(1+\sqrt{2a-3})$

$3a-2=1+1\sqrt{2a-3}+1\sqrt{2a-3}+2a-3$

$-2a+2 \qquad -2a+2$

$a^2=(2\sqrt{2a-3})^2$

$a^2=4(2a-3)$

$a^2=8a-12$

$-8a+12 \quad -8a+12$

$a^2-8a+12=0$

$(a-6)(a-2)=0$

$a-6=0 \quad a-2=0$

$+6 \quad +6 \qquad +2 \quad +2$

$\boxed{a=6,2}$

$a=6$: $\sqrt{3(6)-2}-\sqrt{2(6)-3}=1$

$18-2 \qquad 12-3$

$\sqrt{16}-\sqrt{9}$

$4-3=1$ ✓

$a=2$

$\sqrt{3(2)-2}-\sqrt{2(2)-3}=$

$\sqrt{6-2}-\sqrt{4-3}$

$\sqrt{4}-\sqrt{1}=1$

$2-1=1$ ✓

Rational and Irrational Numbers

In this section we will explore the differences of rational and irrational numbers, and learn several methods for converting between fractions and decimals.

Review: **Rational Numbers**: any # that can be written as a fraction
(decimals repeat or terminate)

Irrational Numbers: any # that cannot be written as a fraction
(decimals that don't repeat or terminate)

Since we will be concentrating on numbers written in decimal form, we will need to know when decimals are rational or irrational.

Break for Practice: Convert the following rationals to decimal form.

1. $\frac{7}{8}$

$$\begin{array}{r} 0.875 \\ 8\overline{)7.000} \\ -64 \\ \hline 60 \\ -56 \\ \hline 40 \\ -40 \\ \hline 0 \end{array}$$

$\boxed{.875}$
terminating decimal

2. $\frac{7}{3}$

$$\begin{array}{r} 2.3\overline{3} \\ 3\overline{)7.00} \\ -6 \\ \hline 10 \\ -9 \\ \hline 10 \\ -9 \\ \hline 1 \end{array}$$

$\boxed{2.3\overline{3}}$
repeating decimal

3. $\frac{9}{11}$

$$\begin{array}{r} 0.81\overline{81} \\ 11\overline{)9.0000} \\ -88 \\ \hline 20 \\ -11 \\ \hline 90 \\ -88 \\ \hline 20 \end{array}$$

$\boxed{0.\overline{81}}$
repeating decimal

4. $\frac{13}{25}$

$$\begin{array}{r} 0.52 \\ 25\overline{)13.00} \\ -125 \\ \hline 50 \\ -50 \\ \hline 0 \end{array}$$

$\boxed{0.52}$
terminating decimal

*note: a repeating decimal will never have more digits in the repeating chunk than the # doing the dividing

Result: 1. repeating and terminating decimals represent rational numbers.

2. All other decimals are irrational.

Break for Practice: Classify each number as rational (R) or irrational (I).

$1.\overline{48}$ R	1.481481148111 … I	$\sqrt{3}+\sqrt{27} \rightarrow \sqrt{3}+3\sqrt{3}$; $27 = 9 \cdot 3$; $3 \cdot 3$; $4\sqrt{3}$ $\boxed{I}$	$\sqrt{3}\cdot\sqrt{27} = \sqrt{81}$; 9 R
$\frac{\pi}{2}$ I	$\sqrt{\frac{25}{64}} = \frac{5}{8}$ R	$\sqrt{\frac{1}{10}} = \frac{1}{\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}} = \frac{\sqrt{10}}{10}$ $\boxed{I}$	

Break for Practice: Convert each decimal into a fraction. Note the different techniques for repeating and terminating decimals.

1. 2.005

$$\frac{2005 \div 5}{1000 \div 5} = \frac{401}{200} = \boxed{2\frac{1}{200}}$$

2. 3.74

$$\frac{374 \div 2}{100 \div 2} = \frac{187}{50}$$

$$\boxed{3\frac{37}{50}}$$

3. 0.0125

$$\frac{125 \div 25}{10000 \div 25} = \frac{5 \div 5}{400 \div 5} = \boxed{\frac{1}{80}}$$

4. $(0.\overline{08} = N)\,100$

$$8.08\cancel{08} = 100N$$
$$-\ .08\cancel{08} \quad -\ N$$
$$\frac{8}{99} = \frac{99N}{99} \qquad N = \boxed{\frac{8}{99}}$$

5. $(0.8\overline{7} = N)\,10$

$$8.777 = 10N$$
$$-\ .877 \quad -\ N$$
$$\frac{7.9}{9} = \frac{9N}{9}$$
$$\frac{7.9}{9} = N$$
$$\boxed{\frac{79}{90} = N}$$

6. $(0.\overline{9} = N)\,10$

$$9.\overline{9} = 10N$$
$$-\ .\overline{9} \quad -\ N$$
$$\frac{9}{9} = \frac{9N}{9}$$
$$\boxed{1 = N}$$

Steps: 1) Let N be the repeating decimal

2) Multiply by a multiple of 10 so one part of the repeating decimal is in front.

3) subtract "N" from both sides

4) solve for N

Extended Practice:

1. Classify each real number or expression as either rational (R), or irrational (I).

$\sqrt{49} = 7$ R	$\sqrt{50} = 5\sqrt{2}$ 25 2 5 5 I	π I	$\frac{22}{7}$ R
$\pi + \frac{1}{\pi}$ I	$\frac{\pi}{1} \cdot \frac{1}{\pi} = \frac{1}{1} = 1$ R	$\sqrt{\frac{1}{2}} + \sqrt{\frac{1}{8}}$ $\frac{1}{\sqrt{2}} + \frac{1}{2\sqrt{2}} =$ I	$\sqrt{\frac{1}{2}} \cdot \sqrt{\frac{1}{8}} = \sqrt{\frac{1}{16}} = \frac{1}{4}$ R
1.23 R	$1.\overline{23}$ R	1.2345678910111213 … I	

2. Write each fraction as a repeating or terminating decimal.

$\frac{5}{8}$ $8\overline{)5.000}$ = 0.625 −48 20 −16 40 −40 0 $\boxed{0.625}$	$\frac{5}{11}$ $11\overline{)5.0000}$ = 0.4545 −44 60 −55 50 −44 60 −55 5 $\boxed{0.\overline{45}}$	$\frac{13}{7}$ $7\overline{)13.000000}$ = $1.\overline{857142}$ −7 * 60 −56 40 −35 50 −49 10 −7 30 −28 20 *6 −14 $\boxed{1.\overline{857142}}$	$\frac{13}{4}$ $4\overline{)13.00}$ = 3.25 −12 10 −8 20 $\boxed{3.25}$

3. Write each decimal as a fraction in lowest terms.

3.004 $\frac{3004 \div 4}{1000 \div 4} = \frac{751}{250}$ $\boxed{3\frac{1}{250}}$	4.72 $4\frac{72}{100}$ or $\frac{472}{100}$ $4\frac{18}{25}$ or $\frac{118}{25}$ $\boxed{4\frac{18}{25}}$	0.1375 $\frac{1375 \div 25}{10000 \div 25} = \frac{55 \div 5}{400 \div 5}$ $\boxed{\frac{11}{80}}$

4. Write each decimal as a fraction in lowest terms.

$(0.\bar{5} = N)10$ $5.\bar{5} = 10N$ $-\ .\bar{5} \quad -N$ $\frac{5}{9} = \frac{9N}{9}$ $\boxed{\frac{5}{9}}$	$(0.\overline{36} = N)100$ $36.\overline{36} = 100N$ $-\ .\overline{36} \quad -N$ $\frac{36}{99} = \frac{99N}{99}$ $\frac{12}{33} = \boxed{\frac{4}{11}}$	$(1.\overline{27} = N)100$ $127.\overline{27} = 100N$ $-\ 1.\overline{27} \quad -N$ $\frac{126}{99} = \frac{99N}{99}$ $\frac{14}{11} = \boxed{1\frac{3}{11}}$

The Imaginary Number, i

In this section we shall see how we can simplify expressions involving square roots of negatives. Up until this time we simply said that they were not real. Instead they are imaginary. Imaginary numbers have applications in electricity, optics, hydrodynamics, etc.

Definition: $i = \sqrt{-1}$ *and* $i^2 = -1$

By extension, we can say that $\sqrt{-r} = i\sqrt{r}$. This allows us to solve and simplify many more algebraic equations and expressions.

Numbers in the form bi, where b does not equal zero, are called pure imaginary.

Break for Practice: * take "i" out of the root first

1. Simplify

$\sqrt{-49}$ $-1 \quad 49$ $7 \quad 7$ $\boxed{7i}$ or $\sqrt{-49}$ $\sqrt{-1} \cdot \sqrt{49}$ $i \cdot 7$ $7i$	$\sqrt{-10}$ $-1 \quad 10$ $\boxed{i\sqrt{10}}$	$\sqrt{-28}$ $-1 \quad 28$ $i \cdot 4 \quad 7$ $2 \quad 2$ $\boxed{2i\sqrt{7}}$ or $i\sqrt{28}$ $4 \quad 7$ $2 \quad 2$ $2i\sqrt{7}$
$-3\sqrt{-144}$ $-3i\sqrt{144}$ $-3i(12)$ $\boxed{-36i}$	$3\sqrt{-12}$ $3i\sqrt{12}$ $4 \quad 3$ $2 \quad 2$ $3i(2)\sqrt{3} = \boxed{6i\sqrt{3}}$	