2. Simplify the product.

$7i \cdot 5i$ $= 35i^2$ $*\ i^2=-1$ $\boxed{-35}$	$(6i)^2$ $6^2 i^2$ $36(-1)$ $\boxed{-36}$	$(2i\sqrt{3})^2$ $(2i)^2(\sqrt{3})^2$ $4i^2 \cdot 3$ $-4 \cdot 3 = \boxed{-12}$
$\sqrt{7} \cdot \sqrt{-14}$ $\sqrt{7} \cdot i\sqrt{14}$ $i\sqrt{98}$ $\boxed{7i\sqrt{2}}$ 49 2 7 7	$\sqrt{-6} \cdot \sqrt{-15}$ $i\sqrt{6} \cdot i\sqrt{15}$ $i^2\sqrt{90}$ $3i^2\sqrt{2 \cdot 5}$ 9 10 $\boxed{-3\sqrt{10}}$ 3 3 2 5	

3. Simplify the sum or difference.

$\sqrt{-20} + \sqrt{-45}$ $i\sqrt{20} + i\sqrt{45}$ 4 5 9 5 2 2 3 3 $2i\sqrt{5} + 3i\sqrt{5}$ $\boxed{5i\sqrt{5}}$	$4\sqrt{-3} - \sqrt{-75}$ $4i\sqrt{3} - i\sqrt{75}$ 25 3 5 5 $4i\sqrt{3} - 5i\sqrt{3}$ $\boxed{-i\sqrt{3}}$

4. Divide. Note: Just like radicals, we can't leave an i in the denominator. $*\ i^2=-1$

$\frac{5}{i} \cdot \frac{i}{i} = \frac{5i}{i^2} = -5i$	$\frac{6}{7i}\left(\frac{i}{i}\right) = \frac{6i}{7i^2} = \boxed{\frac{6i}{-7}}$	$\frac{10}{\sqrt{-5}} = \frac{10}{i\sqrt{5}}\left(\frac{i\sqrt{5}}{i\sqrt{5}}\right) \frac{10i\sqrt{5}}{i^2\sqrt{25}}$ $= \frac{10i\sqrt{5}}{-5}$ $= \boxed{-2i\sqrt{5}}$

5. Solve . * $\pm$

$x^2 + 100 = 0$	$3x^2 + 23 = 5$
$-100 \quad -100$ $\sqrt{x^2} = \sqrt{-100}$ $x = \pm i\sqrt{100}$ $\boxed{x = \pm 10i}$	$-23 \quad -23$ $\frac{3x^2}{3} = \frac{-18}{3}$ $\sqrt{x^2} = \sqrt{-6}$ $\boxed{x = \pm i\sqrt{6}}$

Extended Practice:

1. Simplify

$\sqrt{-81} = i\sqrt{81}$ $9i$	$-4\sqrt{-36}$ $-4i\sqrt{36}$ $-4i(6)$ $\boxed{-24i}$	$\sqrt{-20}$ $i\sqrt{20}$ $4 \quad 5$ $2 \quad 2$ $\boxed{2i\sqrt{5}}$
$3\sqrt{-8}$ $3i\sqrt{8}$ $4 \quad 2$ $2 \quad 2$ $3i \cdot 2\sqrt{2} = \boxed{6i\sqrt{2}}$	$2i \cdot 3i$ $6i^2$ $\boxed{-6}$	$\sqrt{7} \cdot \sqrt{-7}$ $\sqrt{7} \cdot i\sqrt{7}$ $i\sqrt{49}$ $\boxed{7i}$
$\sqrt{-5} \cdot \sqrt{-10}$ $i^2 = -1$ $i\sqrt{5} \cdot i\sqrt{10}$ $i^2\sqrt{50}$ $25 \quad 2$ $5 \quad 5$ $\boxed{-5\sqrt{2}}$	$(7i)^2$ $49i^2$ $\boxed{-49}$	$(-i)^2$ i^2 $\boxed{-1}$
$(i\sqrt{2})^2$ $i^2(\sqrt{2})^2$ $-1(2)$ $\boxed{-2}$	$\frac{8}{3i} \cdot \left(\frac{i}{i}\right) = \frac{8i}{3i^2}$ $= \boxed{\frac{8i}{-3}}$	$\frac{21}{\sqrt{-7}} = \frac{21}{i\sqrt{7}}\left(\frac{i\sqrt{7}}{i\sqrt{7}}\right)$ $\frac{21i\sqrt{7}}{i^2 \cdot 7} = \frac{\cancel{21}i\sqrt{7}}{\cancel{-7}}$ $\boxed{-3i\sqrt{7}}$

2. Simplify

$\sqrt{-25}+\sqrt{-36}$ $i\sqrt{25}+i\sqrt{36}$ $5i+6i$ $\boxed{11i}$	$3\sqrt{-2}-\sqrt{-50}$ $3i\sqrt{2}-i\sqrt{50}$ 25 2 5 5 $3i\sqrt{2}-5i\sqrt{2} = \boxed{-2i\sqrt{2}}$

3. Solve

$x^2+144=0$ $-144 \quad -144$ $\sqrt{x^2}=\sqrt{-144}$ $x=\pm i\sqrt{144}$ $\boxed{x=\pm 12i}$	$3u^2+40=4$ $-40 \quad -40$ $\frac{3u^2}{3}=\frac{-36}{3}$ $\sqrt{u^2}=\sqrt{-12}$ $u=\pm i\sqrt{12}$ 4 3 2 2 $\boxed{u=\pm 2i\sqrt{3}}$

Complex Numbers

In the last section, we saw what pure imaginary numbers look like. Previously, you already knew what real (anything on the number line) numbers look like. If we put these two sets of numbers together, we form the set of complex numbers.

Definition: **Complex Numbers** – numbers in the form $a \pm bi$, where a *and* $b \in$ *reals* .

Examples: $2+3i$ *and* $7-i\sqrt{3}$

Note: $a \pm bi$

pure imaginary part

real part

In this section we will learn how to perform the four basic operations on complex numbers.

Break for Practice:

1. Simplify the sum or difference

* combine real parts together and imaginary parts together

* can't mix

$(7+3i)+(2-5i)$ $9-2i$	$(4-7i)-(5-3i)$ $4-7i-5+3i$ $\boxed{-1-4i}$
$4(1-2i)+3(-7+5i)$ $4-8i-21+15i$ $\boxed{-17+7i}$	$2(-3+i)-5(2-2i)$ $-6+2i-10+10i$ $\boxed{-16+12i}$

2. Simplify the product. * always written with real part first

$2i(4+7i)$ $8i+14i^2$ $8i-14$ $\boxed{-14+8i}$	$-4i(-5+i)$ $20i-4i^2$ $20i+4$ $\boxed{4+20i}$
$(-3+2i)(4+5i)$ $-12-15i+8i+10i^2$ $-12-7i-10$ $\boxed{-22-7i}$	$(3-5i)^2$ $(3-5i)(3-5i)$ $9-15i-15i+25i^2$ $9-30i-25$ $\boxed{-16-30i}$
$(2+3i)(2-3i)$ $4-\cancel{6i}+\cancel{6i}-9i^2$ $4+9=\boxed{13}$	The last example showed the product of complex conjugates. These hold the key for simplifying quotients of complex numbers.

3. Simplify the quotient. * product of complex$(a+bi)$ conjugates is real ie) $(a+bi)(a-bi)$

$\frac{6}{1+3i}\left(\frac{1-3i}{1-3i}\right)=\frac{6-18i}{1-3i+3i-9i^2}$ $=\frac{6-18i}{1+9}=\frac{\overset{3}{\cancel{6}}-\overset{9}{\cancel{18}}i}{\cancel{10}\,5}$ $\frac{3-9i}{5}$ have to separate real and imag. $\boxed{\frac{3}{5}-\frac{9i}{5}}$	$\frac{2+i}{6-2i}\left(\frac{6+2i}{6+2i}\right)=\frac{12+4i+6i+\overset{-2}{\cancel{2i^2}}}{36+\cancel{12i}-\cancel{12i}-\underset{+4}{4i^2}}$ $=\frac{10+10i}{40}$ $=\frac{1+i}{4}$ $=\boxed{\frac{1}{4}+\frac{i}{4}}$

Extended Practice: Simplify

1. $(9+2i)+(1-7i)$ $10-5i$	2. $(5-7i)-(8+2i)$ $5-7i-8-2i$ $\boxed{-3-9i}$
3. $3(-2+i)-4(3-2i)$ $-6+3i-12+8i$ $\boxed{-18+11i}$	4. $i(3+4i)$ $3i+4i^2$ $3i-4$ $\boxed{-4+3i}$
5. $-4i(-2+i)$ $8i-4i^2$ $8i+4$ $\boxed{4+8i}$	6. $(3-i)(3+i)$ $9+\cancel{3i}-\cancel{3i}-i^2$ $9+1=\boxed{10}$
7. $(-4+i)(8+5i)$ $-32-20i+8i+5i^2$ $-32-12i-5$ $\boxed{-37-12i}$	8. $(2-4i)^2$ $(2-4i)(2-4i)$ $4-8i-8i+16i^2$ $4-16i-16$ $\boxed{-12-16i}$
9. $(-1+i\sqrt{3})^2$ $(-1+i\sqrt{3})(-1+i\sqrt{3})$ $1-i\sqrt{3}-i\sqrt{3}+i^2(3)$ $1-2i\sqrt{3}-3 \rightarrow \boxed{-2-2i\sqrt{3}}$	10. $\frac{5}{3+4i}\left(\frac{3-4i}{3-4i}\right)=\frac{15-20i}{9-12i+12i-16i^2}$ $=\frac{15-20i}{9+16}=\frac{\cancel{15}^{3}-\cancel{20}^{4}i}{\cancel{25}_{5}}=\boxed{\frac{3}{5}-\frac{4i}{5}}$
11. $\frac{15}{2-i}\left(\frac{2+i}{2+i}\right)=\frac{30+15i}{4+2i-2i-i^2}=\frac{30+15i}{4+1}$ $\frac{\cancel{30}^{6}+\cancel{15}^{3}i}{\cancel{5}_{1}}$ $\boxed{6+3i}$	12. $\frac{-1-2i}{-1+2i}\left(\frac{-1-2i}{-1-2i}\right)=\frac{1+2i+2i+4i^2}{1+2i-2i-4i^2}$ $=\frac{1+4i-4}{1+4}=\boxed{\frac{-3}{5}+\frac{4i}{5}}$

13. Find the reciprocal of each of the following.

a) $2+3i$: $\frac{1}{2+3i}\left(\frac{2-3i}{2-3i}\right)=\frac{2-3i}{4-\cancel{6i}+\cancel{6i}-9i^2}=\frac{2-3i}{4+9}=\frac{2-3i}{13}=\boxed{\frac{2}{13}-\frac{3i}{13}}$

b) $1-4i$: $\frac{1}{1-4i}\left(\frac{1+4i}{1+4i}\right)=\frac{1+4i}{1+4i-4i-16i^2}=\frac{1+4i}{1+16}=\frac{1+4i}{17}=\boxed{\frac{1}{17}+\frac{4i}{17}}$