4. $\frac{1}{\sqrt[5]{8}}$ $\quad \frac{1}{\sqrt[5]{2^3}} \cdot \left(\frac{\sqrt[5]{2^2}}{\sqrt[5]{2^2}}\right) = \frac{\sqrt[5]{2^2}}{2} = \boxed{\frac{\sqrt[5]{4}}{2}}$

$2\,2\,2$

$\sqrt[5]{2^3}$ $\qquad \sqrt[5]{2^5}$

5. $\frac{2}{\sqrt[4]{125}} \rightarrow \frac{2}{\sqrt[4]{5^3}} \cdot \left(\frac{\sqrt[4]{5}}{\sqrt[4]{5}}\right) \quad \boxed{\frac{2\sqrt[4]{5}}{5}}$

$25\ 5$

$5\ 5$

$\sqrt[4]{5^3}$

Extended Practice: Simplify

1. $\sqrt{3} \cdot \sqrt{15}$ $= \sqrt{45}$ $\sqrt{9}\ \sqrt{5}$ $\boxed{3\sqrt{5}}$	2. $\sqrt{6} \cdot \sqrt{8}$ $= \sqrt{48}$ $\sqrt{16} \cdot \sqrt{3}$ $\boxed{4\sqrt{3}}$
3. $\sqrt[3]{25} \cdot \sqrt[3]{10}$ $= \sqrt[3]{250} = \sqrt[3]{5^3 \cdot 2}$ $25\ \ 10$ $5\,5\ \ 5\,2$ $\boxed{5\sqrt[3]{2}}$	4. $\sqrt[3]{8} \cdot \sqrt[3]{8}$ $\quad$ or $\rightarrow$ $\sqrt[3]{64} = \sqrt[3]{2^6}$ $2 \cdot 2$ 4 $8\ 8$ $2\,2\,2\ \ 2\,2\,2$ $2^2 = \boxed{4}$
5. $(7\sqrt{2})^2$ $7^2(\sqrt{2})^2$ $49 \cdot 2 = \boxed{98}$	6. $(5\sqrt{3})^2$ $5^2(\sqrt{3})^2$ $25 \cdot 3 = \boxed{75}$
7. $(2\sqrt{2})(5\sqrt{8})$ $10\sqrt{16}$ $10 \cdot 4 = \boxed{40}$	8. $\frac{7}{\sqrt{2}} \cdot \left(\frac{\sqrt{2}}{\sqrt{2}}\right)$ $\boxed{\frac{7\sqrt{2}}{2}}$

9. $\frac{6}{\sqrt{3}} \cdot \left(\frac{\sqrt{3}}{\sqrt{3}}\right) = \frac{6\sqrt{3}}{3} = \boxed{2\sqrt{3}}$ $\sqrt{9} = 3$	10. $\frac{15}{\sqrt{5}} \cdot \left(\frac{\sqrt{5}}{\sqrt{5}}\right) = \frac{15\sqrt{5}}{5} = \boxed{3\sqrt{5}}$ $\sqrt{25} = 5$
11. $\frac{3}{\sqrt[4]{8}} \cdot \left(\frac{\sqrt[4]{2}}{\sqrt[4]{2}}\right) = \frac{3\sqrt[4]{2}}{\sqrt[4]{2^4}} = \boxed{\frac{3\sqrt[4]{2}}{2}}$ 2 2 2 $\sqrt[4]{2^3}$	12. $\frac{8}{\sqrt[3]{25}} \cdot \left(\frac{\sqrt[3]{5}}{\sqrt[3]{5}}\right) = \frac{8\sqrt[3]{5}}{\sqrt[3]{5^3}} = \boxed{\frac{8\sqrt[3]{5}}{5}}$ 5 5 $\sqrt[3]{5^2}$
13. $\frac{6}{\sqrt[5]{8}} \cdot \left(\frac{\sqrt[5]{2^2}}{\sqrt[5]{2^2}}\right) = \frac{6\sqrt[5]{4}}{\sqrt[5]{2^5}} = \frac{6\sqrt[5]{4}}{2}$ 2 2 2 $\sqrt[5]{2^3}$ $\boxed{3\sqrt[5]{4}}$	14. $\frac{12}{\sqrt[4]{9}} \cdot \left(\frac{\sqrt[4]{3^2}}{\sqrt[4]{3^2}}\right) = \frac{12\sqrt[4]{3^2}}{\sqrt[4]{3^4}} = \frac{12\sqrt[4]{9}}{3}$ $\sqrt[4]{3^2}$ $= \boxed{4\sqrt[4]{9}}$

Sums and Differences of Radicals

In this section we will see how to deal with expressions involving the sums and differences of radicals. In many ways, it is similar to combining like terms with algebraic expressions. Radicals can be added or subtracted when their indices are identical and so are their radicands. $*\sqrt[n]{\underline{a}} + *\sqrt[n]{\underline{a}}$

Break for Practice: Simplify

1. $\sqrt{27} + \sqrt{12}$

$\sqrt{9}\sqrt{3}\ \ \sqrt{4}\sqrt{3}$

$\underline{3}\sqrt{3} + \underline{2}\sqrt{3}$

$\boxed{5\sqrt{3}}$

2. $\sqrt{28} + \sqrt{63}$

$\sqrt{4}\sqrt{7}\ \ \sqrt{9}\sqrt{7}$

$2\sqrt{7} + 3\sqrt{7}$

$5\sqrt{7}$

3. $\sqrt{32} - \sqrt{50} + \sqrt{98}$

$\sqrt{16}\sqrt{2}\ \ \sqrt{25}\sqrt{2}\ \ \sqrt{49}\sqrt{2}$

$\underline{4\sqrt{2} - 5\sqrt{2}} + 7\sqrt{2}$

$-\sqrt{2} + 7\sqrt{2}$

$\boxed{6\sqrt{2}}$

4. $\sqrt[3]{54} - \sqrt[3]{16} + \sqrt[3]{27}$

9 6   4 4   9 3

3 3 3 2   2 2 2 2   3 3

$\sqrt[3]{3^3 \cdot 2} - \sqrt[3]{2^4} + \sqrt[3]{3^3}$

$3\sqrt[3]{2} - 2\sqrt[3]{2} + 3$

$\boxed{\sqrt[3]{2} + 3}$

5. $\sqrt[3]{135} - \sqrt[3]{40} + \sqrt[3]{2}$

45 3   8 5

9 5   2 2 2

3 3

$\sqrt[3]{3^3 \cdot 5} - \sqrt[3]{2^3 \cdot 5} + \sqrt[3]{2}$

$3\sqrt[3]{5} - 2\sqrt[3]{5} + \sqrt[3]{2} \rightarrow \boxed{\sqrt[3]{5} + \sqrt[3]{2}}$

6. $\sqrt{2}(\sqrt{32} + \sqrt{12})$

$\sqrt{64} + \sqrt{24}$

$\sqrt{4}\sqrt{6}$

$8 + 2\sqrt{6}$

$\boxed{8 + 2\sqrt{6}}$

7. $2\sqrt{3}(4\sqrt{2} - 5\sqrt{12})$

$8\sqrt{6} - 10\sqrt{36}$

$8\sqrt{6} - 10 \cdot 6$

$\boxed{8\sqrt{6} - 60}$

Extended Practice: Simplify

1. $\sqrt{50} + \sqrt{18}$ $\sqrt{25}\sqrt{2}$ $\sqrt{9}\sqrt{2}$ $5\sqrt{2} + 3\sqrt{2} = \boxed{8\sqrt{2}}$	2. $3\sqrt{12} - \sqrt{48}$ $\sqrt{4}\sqrt{3}$ $\sqrt{16}\sqrt{3}$ $3 \cdot 2\sqrt{3} - 4\sqrt{3}$ $6\sqrt{3} - 4\sqrt{3} = \boxed{2\sqrt{3}}$
3. $\sqrt{27} + 2\sqrt{75}$ $\sqrt{9}\sqrt{3}$ $\sqrt{25}\sqrt{3}$ $3\sqrt{3} + 2 \cdot 5\sqrt{3}$ $3\sqrt{3} + 10\sqrt{3} = \boxed{13\sqrt{3}}$	4. $5\sqrt{2} - 2\sqrt{5}$ $\boxed{5\sqrt{2} - 2\sqrt{5}}$
5. $\sqrt{50} + \sqrt{63} - \sqrt{32}$ $\sqrt{25}\sqrt{2}$ $\sqrt{9}\sqrt{7}$ $\sqrt{16}\sqrt{2}$ $5\sqrt{2} + 3\sqrt{7} - 4\sqrt{2} = \boxed{3\sqrt{7} + \sqrt{2}}$	6. $\sqrt{18} + \sqrt{24} - \sqrt{54}$ $\sqrt{9}\sqrt{2}$ $\sqrt{4}\sqrt{6}$ $\sqrt{9}\sqrt{6}$ $3\sqrt{2} + 2\sqrt{6} - 3\sqrt{6} = \boxed{3\sqrt{2} - \sqrt{6}}$

7. $\sqrt[3]{54}+\sqrt[3]{40}+\sqrt[3]{16}$ 9 6 8 5 4 4 3 3 3 2 2 2 2 2 2 2 2 $\sqrt[3]{3^3\cdot 2}+\sqrt[3]{2^3\cdot 5}+\sqrt[3]{2^4}$ $3\sqrt[3]{2}+2\sqrt[3]{5}+2\sqrt[3]{2} = \boxed{5\sqrt[3]{2}+2\sqrt[3]{5}}$	8. $\sqrt{2}(\sqrt{8}+\sqrt{10}) = \sqrt{16}+\sqrt{20}$ $4+2\sqrt{5}$ $\sqrt{4}\sqrt{5}$ $\boxed{4+2\sqrt{5}}$
9. $\sqrt{3}(\sqrt{12}-\sqrt{24})$ $\sqrt{36}-\sqrt{72}$ $6-\sqrt{36}\sqrt{2}$ $\boxed{6-6\sqrt{2}}$	10. $3\sqrt{5}(\sqrt{5}+2\sqrt{75})$ $3\sqrt{25}+6\sqrt{375}$ $3\cdot 5+6\cdot\sqrt{25}\sqrt{15}$ $15+6\cdot 5\sqrt{15}$ $\boxed{15+30\sqrt{15}}$

Binomials Containing Radicals

In this section we will see how we can multiply and divide binomials that contain radicals. The process is very similar to what we used with algebraic binomials in the past.

How do you think you would multiply something like this? F.O.I.L

$(2+\sqrt{5})(3-\sqrt{5})$ $6-2\sqrt{5}+3\sqrt{5}-5 = 1+\sqrt{5}$

Break for Practice: Simplify

1. $(3+\sqrt{2})(\sqrt{3}-4)$

$\boxed{3\sqrt{3}-12+\sqrt{6}-4\sqrt{2}}$

2. $(2\sqrt{5}+3)(\sqrt{5}-4)$

$2\sqrt{25}-8\sqrt{5}+3\sqrt{5}-12$

$2\cdot 5$

$10-5\sqrt{5}-12$

$\boxed{-2-5\sqrt{5}}$

3. $(\sqrt{6}-2)^2$

$(\sqrt{6}-2)(\sqrt{6}-2)$

$\sqrt{36}-2\sqrt{6}-2\sqrt{6}+4$

$6-4\sqrt{6}+4$

$\boxed{2-4\sqrt{6}}$

4. $(2+\sqrt{3})(2-\sqrt{3})$

$4-\cancel{2\sqrt{3}}+\cancel{2\sqrt{3}}-\sqrt{9}$

$4-3$

$\boxed{1}$

5. $(1+\sqrt{5})(1-\sqrt{5})$

$1-\cancel{\sqrt{5}}+\cancel{\sqrt{5}}-\sqrt{25}$

$1-5 = \boxed{-4}$

$(a+\sqrt{b})(a-\sqrt{b})$

The last two examples illustrate multiplying conjugates. Conjugates are identical binomials except for the middle sign. Note that when these are multiplied, the radical disappears. This idea can be used to rationalize denominators in the following problems.

mult by conjugate

Break for Practice: Simplify

1. $\frac{5}{(2-\sqrt{3})}\cdot\left(\frac{2+\sqrt{3}}{2+\sqrt{3}}\right) = \frac{10+5\sqrt{3}}{4+\cancel{2\sqrt{3}}-\cancel{2\sqrt{3}}-\sqrt{9}} = \frac{10+5\sqrt{3}}{1}$

   $4 \quad -3$

   $\boxed{10+5\sqrt{3}}$

2. $\frac{3}{(\sqrt{5}+\sqrt{3})}\cdot\left(\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}}\right) = \frac{3\sqrt{5}-3\sqrt{3}}{\sqrt{25}-\cancel{\sqrt{15}}+\cancel{\sqrt{15}}-\sqrt{9}} = \boxed{\frac{3\sqrt{5}-3\sqrt{3}}{2}}$

   $5 \quad -3$

Extended Practice: Simplify

1. $(3+\sqrt{7})(3-\sqrt{7})$ $9-\cancel{3\sqrt{7}}+\cancel{3\sqrt{7}}-\sqrt{49}$ $9-7 = \boxed{2}$	2. $(\sqrt{7}+1)^2 = (\sqrt{7}+1)(\sqrt{7}+1)$ $\sqrt{49}+\sqrt{7}+\sqrt{7}+1$ $7+2\sqrt{7}+1$ $\boxed{8+2\sqrt{7}}$
3. $(6-\sqrt{3})(4+\sqrt{3})$ $24+6\sqrt{3}-4\sqrt{3}-\sqrt{9}$ $24+2\sqrt{3}-3$ $\boxed{21+2\sqrt{3}}$	4. $\frac{1}{(4-\sqrt{3})}\cdot\left(\frac{4+\sqrt{3}}{4+\sqrt{3}}\right) = \frac{4+\sqrt{3}}{16+\cancel{4\sqrt{3}}-\cancel{4\sqrt{3}}-\sqrt{9}}$ -3 $\boxed{\frac{4+\sqrt{3}}{13}}$
5. $\frac{1}{6+\sqrt{3}}\cdot\left(\frac{6-\sqrt{3}}{6-\sqrt{3}}\right) = \frac{6-\sqrt{3}}{36-\cancel{6\sqrt{3}}+\cancel{6\sqrt{3}}-\sqrt{9}}$ -3 $\boxed{\frac{6-\sqrt{3}}{33}}$	6. $(3+4\sqrt{3})(2-\sqrt{3})$ $6-3\sqrt{3}+8\sqrt{3}-4\sqrt{9}$ $6+5\sqrt{3}-12$ $\boxed{-6+5\sqrt{3}}$
7. $\frac{3}{\sqrt{5}+\sqrt{2}}\cdot\left(\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}}\right) = \frac{3\sqrt{5}-3\sqrt{2}}{\sqrt{25}-\cancel{\sqrt{10}}+\cancel{\sqrt{10}}-\sqrt{4}}$ $5 \quad -2$ $\frac{\cancel{3}\sqrt{5}-\cancel{3}\sqrt{2}}{\cancel{3}} = \boxed{\sqrt{5}-\sqrt{2}}$	8. $\frac{(\sqrt{5}+1)}{\sqrt{5}-3}\cdot\left(\frac{\sqrt{5}+3}{\sqrt{5}+3}\right) = \frac{\sqrt{25}+3\sqrt{5}+\sqrt{5}+3}{\sqrt{25}+3\sqrt{5}-3\sqrt{5}-9}$ $= \frac{5+4\sqrt{5}+3}{5-9}$ $= \frac{\cancel{8}^{-2}+\cancel{4}^{-1}\sqrt{5}}{\cancel{-4}} = \boxed{-2-\sqrt{5}}$

Solving Equations Containing Radicals

In this section we will learn how to solve radical equations. Radical equations include a variable under a radical sign.

An example of a radical equation is the formula for calculating braking distance for a moving car. The formula is $s = \sqrt{22d}$. s is the speed in miles per hour, and d is the braking distance in feet.

a) What was the speed of a car that left skid marks 160 feet long?

$s = \sqrt{22(160)}$ $\quad s \approx 59.3$ mph

$s = \sqrt{3520}$

b) What braking distance is needed for stopping a car travelling 65 mph?

$(65)^2 = (\sqrt{22d})^2$

$\frac{4225}{22} = \frac{22d}{22}$ $\quad d \approx 192.05$ ft

This shows an example of solving a very simple radical equation. In general, these are the steps for solving radical equations.

Steps:

1. isolate a radical term.
2. Undo the radical (taking the n^{th} power)
3. repeat if necessary until all radicals are gone.
4. solve the resulting equation.
5. check solutions to eliminate any extraneous solutions.

Break for Practice: Solve each radical equation and identify any extraneous solutions.

1. $(\sqrt{x-5})^2 = 5^2$

$x - 5 = 25$

$+5 \quad +5$

$\boxed{x = 30}$

check $\sqrt{30-5} = 5$

$\sqrt{25} = 5$ ✓

2. $(\sqrt{3x-5})^2 = 4^2$

$3x - \cancel{5} = 16$

$+\cancel{5} \quad +5$

$\frac{3x}{3} = \frac{21}{3}$

$\boxed{x = 7}$

check

$\sqrt{3(7)-5} = 4$

$\sqrt{21-5} = 4$

$\sqrt{16} = 4$ ✓