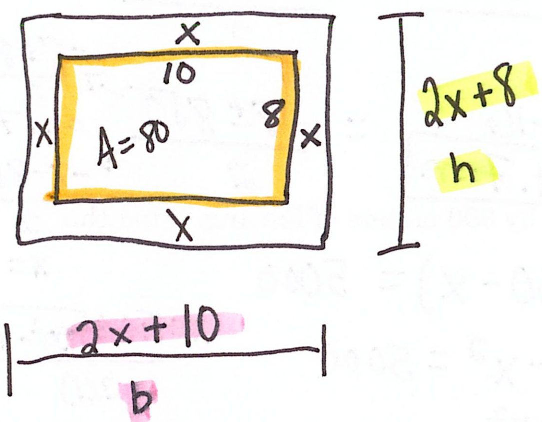
Applications with Quadratic Equations

Now it's time to see where quadratic equations might be used.

Break for Practice: Solve

1. A flower garden with dimensions of 8 meters by 10 meters is enclosed by a walkway of uniform width. If the area of the walkway is 40 square meters, then what is the width of the path?



$$A_{path} = A_{big} - A_{small\ (garden)}$$

$$40 = (\underbrace{2x+8}_{h})(\underbrace{2x+10}_{b}) - 80$$

$$40 = 4x^2 + 20x + 16x + \cancel{80} - \cancel{80}$$

$$-40 \qquad\qquad -40$$

$$0 = 4x^2 + 36x - 40$$

$$a=4 \quad b=36 \quad c=-40$$

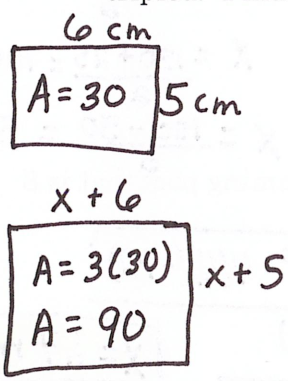
$$x = \frac{-36 \pm \sqrt{(36)^2 - 4(4)(-40)}}{2(4)}$$

$$= \frac{-36 \pm \sqrt{1936}}{8}$$

$$\frac{-36 \pm 44}{8} \rightarrow \frac{-36+44}{8} \quad \boxed{x = 1m}$$

$$\frac{-36-44}{8} = \cancel{-10}$$ can't be neg.

2. A rectangle is 6 cm long and 5 cm wide. When each dimension is increased by x cm, the area is tripled. Find the value of x.



$$90 = (x+6)(x+5)$$

$$90 = x^2 + 5x + 6x + 30$$

$$-90 \qquad\qquad -90$$

$$0 = x^2 + 11x - 60$$

$$a=1 \quad b=11 \quad c=-60$$

$$x = \frac{-11 \pm \sqrt{11^2 - 4(1)(-60)}}{2(1)}$$

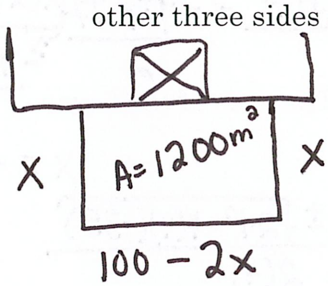
$$= \frac{-11 \pm \sqrt{361}}{2}$$

$$= \frac{-11 \pm 19}{2} \rightarrow \frac{-11+19}{2} = 4$$

$$\frac{-11-19}{2} = \cancel{-15}$$

$$\boxed{x = 4\text{ cm}}$$

3. A rectangular animal pen with an area of 1200 square meters has one side along a barn. The other three sides are enclosed by 100 meters of fencing. Find the dimensions of the pen.



$$A = b \cdot h$$

$$1200 = (100-2x)x$$

$$1200 = 100x - 2x^2$$

$$-1200 \qquad\qquad -1200$$

$$0 = -2x^2 + 100x - 1200$$

$$a=-2$$

$$b=100$$

$$c=-1200$$

$$x = \frac{-100 \pm \sqrt{100^2 - 4(-2)(-1200)}}{2(-2)}$$

$$= \frac{-100 \pm \sqrt{400}}{-4}$$

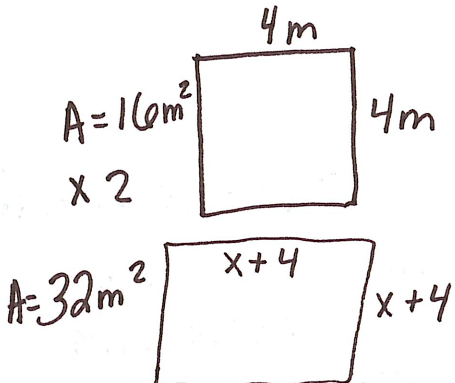
$$= \frac{-100 \pm 20}{-4} \rightarrow \frac{-100+20}{-4} = 20$$

$$\frac{-100-20}{-4} = 30$$

x=20 20m x 60m or x=30 30m x 40m

Extended Practice: Solve

1. Each side of a square is 4 meters long. When each side is increased by x meters, the area is doubled. Find the value of x.



$32 = (x+4)(x+4)$

$32 = x^2+4x+4x+16$

$-32 \qquad\qquad -32$

$0 = x^2+8x-16$

$a=1 \quad b=8 \quad c=-16$

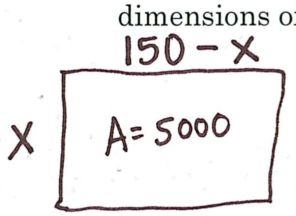
$\boxed{x = 1.7\text{ m}}$

$x = \dfrac{-8 \pm \sqrt{8^2-4(1)(-16)}}{2(1)}$

$= \dfrac{-8 \pm \sqrt{128}}{2}$

$= \dfrac{-8 \pm 8\sqrt{2}}{2}$ → $\dfrac{-8+8\sqrt{2}}{2} = 1.7$ → $\dfrac{-8-8\sqrt{2}}{2}$ $x = -9.\cancel{7}$

2. A rectangular field with area 5000 square meters is enclosed by 300 meters of fencing. Find the dimensions of the field.



width = x

$\dfrac{300}{2} = \dfrac{2l}{2} + \dfrac{2w}{2}$

$150 = l + w$

$-l \quad -l$

$150 - l = w$

$150-100 = 50$

$150-50 = 100$

$\boxed{50\text{m} \times 100\text{m}}$

$x(150-x) = 5000$

$150x - x^2 = 5000$

$-150x + x^2$

$0 = x^2 - 150x + 5000$

$a = 1 \quad b = -150 \quad c = 5000$

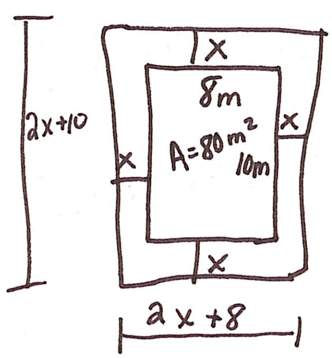
$x = \dfrac{150 \pm \sqrt{150^2-4(1)5000}}{2(1)}$

$= \dfrac{150 \pm \sqrt{2500}}{2}$

$x = \dfrac{150+50}{2} = 100$

$x = \dfrac{150-50}{2} = 50$

3. A walkway of uniform width has area 72 meters squared and surrounds a swimming pool that is 8 meters wide and 10 meters long. Find the width of the walkway.



Area of Walkway $= A_{entire} - A_{pool}$

$72 = (2x+10)(2x+8) - 80$

$72 = 4x^2+16x+20x+\cancel{80}-\cancel{80}$

$-72 \qquad\qquad -72$

$0 = 4x^2 + 36x - 72$

$a=4 \quad b=36 \quad c=-72$

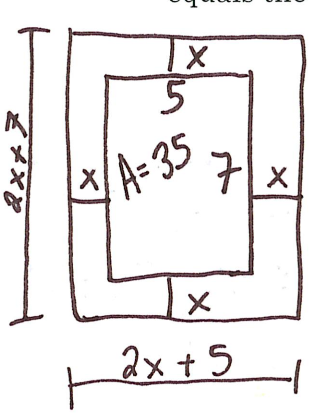
$x = \dfrac{-36 \pm \sqrt{36^2-4(4)(-72)}}{2(4)}$

$= \dfrac{-36 \pm \sqrt{2448}}{8}$

$= \dfrac{-\cancel{36}^{9} \pm \cancel{12}^{3}\sqrt{17}}{\cancel{8}_{2}} = \dfrac{-9 \pm 3\sqrt{17}}{2}$ → $\dfrac{-9+3\sqrt{17}}{2}$ $\boxed{x \approx 1.7}$ → $\dfrac{-9-3\sqrt{17}}{2}$ $x = -10.7$

$\boxed{x = 1.7\text{ m}}$

4. A 5 inch by 7inch photograph is surrounded by a frame of uniform width. The area of the frame equals the area of the photograph. Find the width of the frame.



$\text{Area}_{frame} = A_{entire} - A_{picture}$

$35 = (2x+5)(2x+7) - 35$

$35 = 4x^2+14x+10x+\cancel{35}-\cancel{35}$

$-35 \qquad\qquad -35$

$0 = 4x^2+24x-35$

$a=4$

$b=24$

$c=-35$

$\boxed{x = 1.2\text{ in}}$

$x = \dfrac{-24 \pm \sqrt{24^2-4(4)(-35)}}{2(4)}$

$= \dfrac{-24 \pm \sqrt{1136}}{8}$

$= \dfrac{-24 \pm 33.7}{8}$ → $\dfrac{-24+33.7}{8}$ $\boxed{x = 1.2}$

$\dfrac{-24-33.7}{8} = -\cancel{7.2}$

The Discriminant

In certain situations, it is not necessary to actually solve a quadratic equation, but it is useful to know what kind of solutions (roots) it has. To do this, you need to be able to use the discriminant test.

Consider the following four problems. Each one gives a different type of solution. Solve each with the quadratic formula, and then try to figure out what part of the quadratic formula determines the nature of the solutions.

$2x^2 - 8x + 8 = 0$ a b c $x = \frac{8 \pm \sqrt{8^2 - 4(2)(8)}}{2(2)}$ $= \frac{8 \pm \sqrt{0}}{4}$ $= \frac{8}{4} \pm 0 \rightarrow \frac{8+0}{2} = 4$ $\frac{8-0}{4} = 4$ $x = 2$ one real rational (double root)	$x^2 - x - 12 = 0$ $a=1$ $b=-1$ $c=-12$ $x = \frac{1 \pm \sqrt{(-1)^2 - 4(1)(-12)}}{2(1)}$ $= \frac{1 \pm \sqrt{49}}{2}$ $= \frac{1 \pm 7}{2} \rightarrow \frac{1+7}{2} = 4$ $\frac{1-7}{2} = -3$ $x = 4, -3$ 2 real rational
$x^2 - 2x + 2 = 0$ $a=1$ $b=-2$ $c=2$ $x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)}$ $= \frac{2 \pm \sqrt{-4}}{2}$ $= \frac{2 \pm 2i}{2}$ $x = \boxed{1 \pm i}$ 2 imaginary	$x^2 - 4x - 3 = 0$ $a=1$ $b=-4$ $c=-3$ $x = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(-3)}}{2(1)}$ $= \frac{4 \pm \sqrt{28}}{2}$ $= \frac{4 \pm 2\sqrt{7}}{2}$ 2 real irrational $\boxed{x = 2 \pm \sqrt{7}}$

Result: The discriminant, D = $b^2 - 4ac$

a) If $D < 0$, then there are 2 imaginary solutions.

b) If $D = 0$, then there is one real (double) solution

c) If $D > 0$ and a perfect square, then there are 2 real rational solutions

d) If $D > 0$ but not a perfect square, then there are 2 real irrational solution.

Break for Practice:

1. Use the discriminant to identify the type of solutions for each equation.

$x^2 + 10x + 25 = 0$ $a=1$ $b=10$ $c=25$ $D = 10^2 - 4(1)(25)$ $= 100 - 100$ $D = 0$ $\boxed{\text{One real (double) root}}$	$5x^2 - 6x + 2 = 0$ $a=5$ $b=-6$ $c=2$ $D = (-6)^2 - 4(5)(2)$ $= 36 - 40$ $D = -4$ $\boxed{\text{2 imaginary}}$
$3x^2 - 4x - 7 = 0$ $a=3$ $b=-4$ $c=-7$ $D = (-4)^2 - 4(3)(-7)$ $= 16 + 84$ $D = 100$ $\boxed{\text{2 real rational solutions}}$	$\frac{x^2}{6} + 1 = x$ $-x \quad -x$ $\frac{1}{6}x^2 - x + 1 = 0$ $a = \frac{1}{6}$ $b=-1$ $c=1$ $D = (-1)^2 - \frac{4}{1}\left(\frac{1}{6}\right)\left(\frac{1}{1}\right)$ $= 1 - \frac{4}{6}$ $= 1 - \frac{2}{3}$ $D = \frac{1}{3}$ $\boxed{\text{2 real irrational sol}^{\underline{n}}}$

2. Find the value(s) of k for which the equation $2x^2 + 4x + k = 0$ has the following.

a) One real double root.

$a=2$
$b=4$
$c=k$

$$0 = 4^2 - 4(2)(k)$$
$$0 = 16 - 8k$$
$$-16 \quad -16$$
$$\frac{-16}{-8} = \frac{-8k}{-8}$$
$$\boxed{k=2}$$

b) Two different real roots

$a=2$
$b=4$
$c=k$

$$4^2 - 4(2)(k) > 0$$
$$16 - 8k > 0$$
$$-16 \quad -16$$
$$\frac{-8k}{-8} > \frac{-16}{-8}$$
$$\boxed{k<2}$$

c) Two imaginary roots

$a=2$
$b=4$
$c=k$

$$4^2 - 4(2)(k) < 0$$
$$\cancel{16} - 8k < 0$$
$$-\cancel{16} \quad -16$$
$$\frac{-8k}{-8} < \frac{-16}{-8}$$
$$\boxed{k>2}$$

Extended Practice:

1. Use the discriminant to identify the type of solutions for each equation. $*D = b^2 - 4ac$

$x^2+3x-9=0$ $a=1\ b=3\ c=-9$ $D=3^2-4(1)(-9)$ $=9+36$ $D=45$ 2 real irrational sol$^{\underline{n}}$	$x^2-4x-5=0$ $a=1\ b=-4\ c=-5$ $D=(-4)^2-4(1)(-5)$ $=16+20$ $D=36$ 2 real rational sol$^{\underline{n}}$	$t^2+8t+20=0$ $a=1\ b=8\ c=20$ $D=8^2-4(1)(20)$ $=64-80$ $D=-16$ 2 imaginary sol$^{\underline{n}}$	$3m^2-8m-5=0$ $a=3\ b=-8\ c=-5$ $D=(-8)^2-4(3)(-5)$ $=64+60$ $D=124$ 2 real irrational sol$^{\underline{n}}$
$2y^2-9y+3=0$ $a=2\ b=-9\ c=3$ $D=(-9)^2-4(2)(3)$ $=81-24$ $D=57$ 2 real irrational sol$^{\underline{n}}$	$5t^2-4t+3=0$ $a=5\ b=-4\ c=3$ $D=(-4)^2-4(5)(3)$ $=16-60$ $D=-44$ 2 imaginary sol$^{\underline{n}}$	$z^2+\frac{5}{4}=z$ $-z\quad -z$ $z^2-z+\frac{5}{4}=0$ $a=1\ b=-1\ c=\frac{5}{4}$ $D=(-1)^2-\frac{4}{1}\left(\frac{5}{4}\right)(1)$ $D=1-5$ $D=-4$ 2 imaginary sol$^{\underline{n}}$	$\frac{r^2}{4}+1=r$ $-r\quad -r$ $\frac{1}{4}r^2-r+1=0$ $a=\frac{1}{4}\ b=-1\ c=1$ $D=(-1)^2-\frac{4}{1}\left(\frac{1}{4}\right)(1)$ $=1-1$ $D=0$ one real (double) root

factor, complete the square, quadratic formula

2. Solve each equation using whichever method seems easiest to you.

a) $x^2-6x+5=0$

$(x-5)(x-1)=0$

$x-5=0 \quad x-1=0$

$\boxed{x=5,1}$

b) $y^2+2y-24=0$

$\left(\frac{2}{2}\right)^2$

$(1)^2$

1

$+24 \quad +24$

$y^2+2y+1=24+1$

$\sqrt{(y+1)^2}=\sqrt{25}$

$y+1=\pm 5$

$-1 \quad -1$

$y=-1\pm 5$

$-1+5=4$

$-1-5=-6$

$\boxed{x=-6,4}$

c) $\frac{5(x+7)^2}{5}=\frac{0}{5}$

$\sqrt{(x+7)^2}=\sqrt{0}$

$x+7=0$

$-7 \quad -7$

$\boxed{x=-7}$

d) $\frac{5(x+7)^2}{5}=\frac{25}{5}$

$\sqrt{(x+7)^2}=\sqrt{5}$

$x+7=\pm\sqrt{5}$

$-7 \quad -7$

$\boxed{x=-7\pm\sqrt{5}}$

e) $(2x+5)(X-3)=0$

$2x+5=0$ $\quad$ $x-3=0$

$-5\ \ -5$ $\quad$ $+3\ \ +3$

$\frac{2x}{2}=\frac{-5}{2}$

$\boxed{x=-\frac{5}{2},\ 3}$

3. Find the value(s) of k for which the equation $3x^2-6x-k=0$ has the following.

d) One real double root.

$a=3$

$b=-6$

$c=-k$

$0=(-6)^2-4(3)(-k)$

$0=36+12k$

$-36\ \ -36$

$\frac{-36}{12}=\frac{12k}{12}$

$\boxed{k=-3}$

e) Two different real roots

$a=3$

$b=-6$

$c=-k$

$(-6)^2-4(3)(-k)>0$

$36+12k>0$

$-36\ \ \ -36$

$\frac{12k}{12}>\frac{-36}{12}$

$\boxed{k>-3}$

f) Two imaginary roots

$a=3$

$b=-6$

$c=-k$

$(-6)^2-4(3)(-k)<0$

$36+12k<0$

$-36\ \ \ -36$

$\frac{12k}{12}<\frac{-36}{12}$

$\boxed{k<-3}$

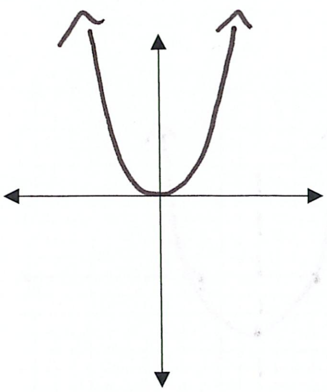
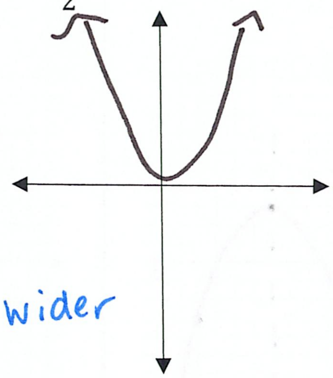
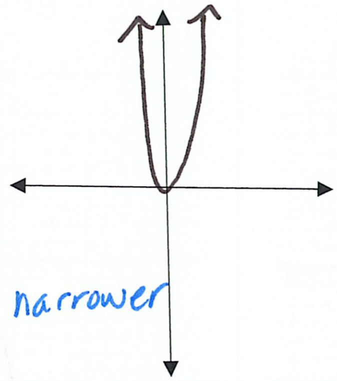
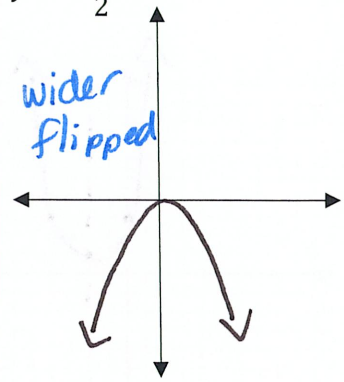
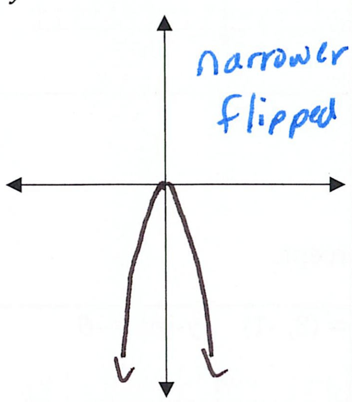
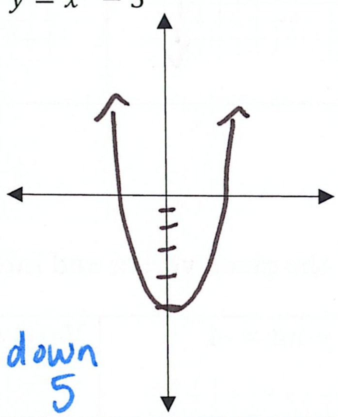
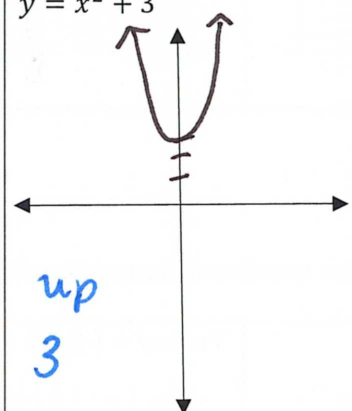
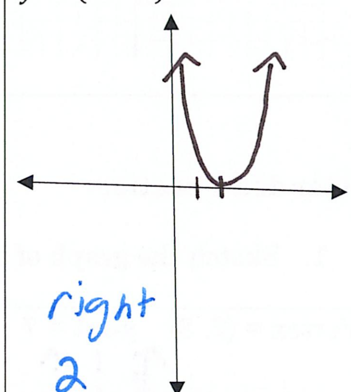
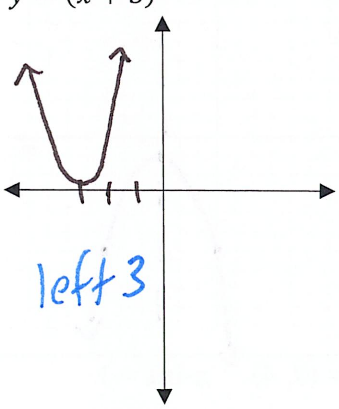
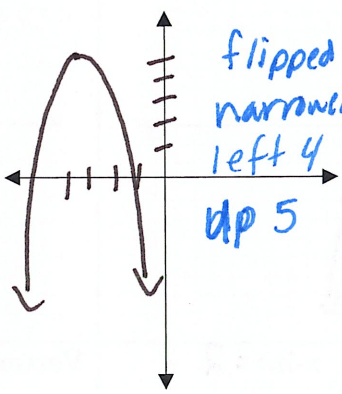
Introduction to Graphs of Quadratic Functions

In this section we will begin exploring the graphs of quadratics. We will use graphing calculators/computers in this section, and in future sections we will learn useful techniques that we can do by hand.

Definition: **Quadratic Functions** are functions in the form

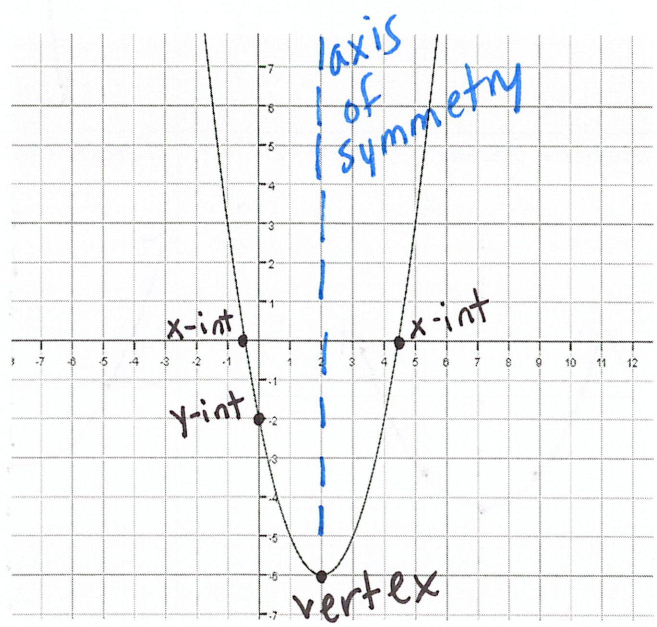
$$y=ax^2+bx+c.\quad a, b, \text{and } c \text{ are constants but } a \neq 0$$

Use the graphing calculator/computer to make quick sketches for the following.

$y = x^2$ 	$y = \frac{1}{2}x^2$  wider	$y = 2x^2$  narrower	$y = -\frac{1}{2}x^2$  wider flipped
$y = -3x^2$  narrower flipped	$y = x^2 - 5$  down 5	$y = x^2 + 3$  up 3	$y = (x-2)^2$  right 2
$y = (x+3)^2$  left 3	$y = -3(x+4)^2 + 5$  flipped narrower left 4 up 5		

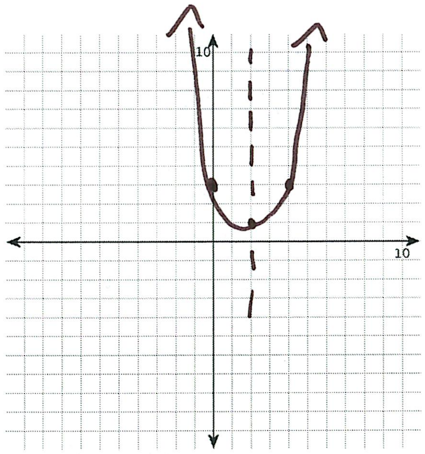
The shape of these graphs is called a parabola.

Label the parts in the graph below.

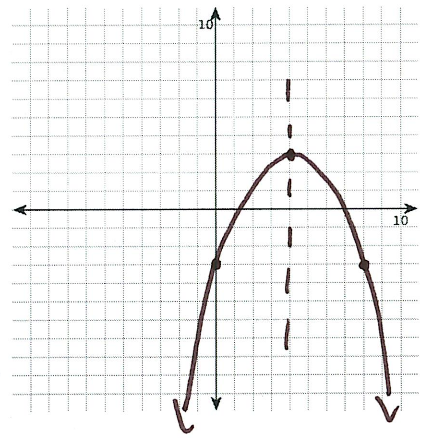


Break for Practice: Quickly sketch parabolas from the following information.

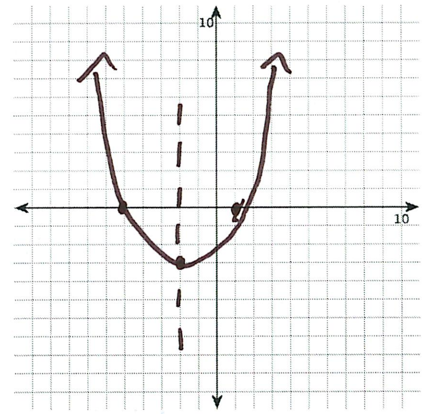
Vertex = (2, 1)   y – int = 3



Vertex = (4, 3)   y – int = -3



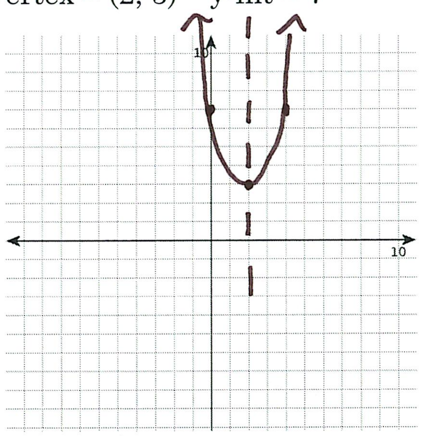
Vertex = (-2, -3)   x – int = 1



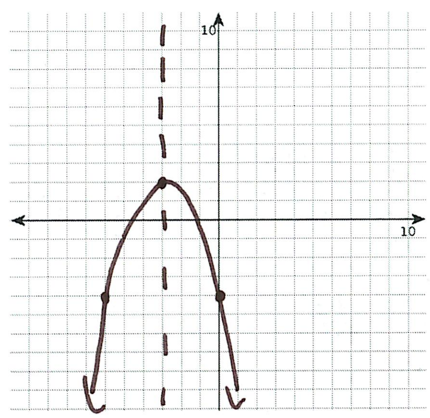
Extended Practice:

1. Sketch the graph of the quadratic function with the given vertex and intercept.

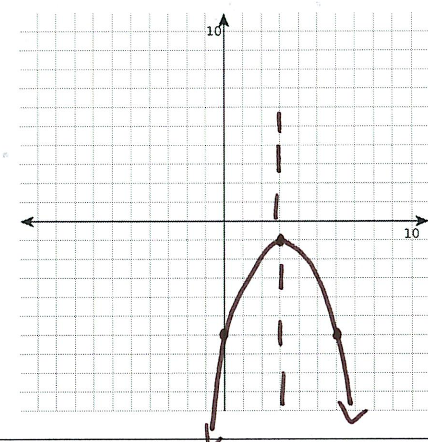
Vertex = (2, 3)   y-int = 7



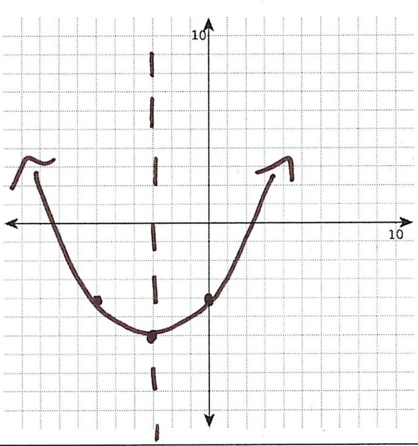
Vertex = (-3, 2)   y-int = -4



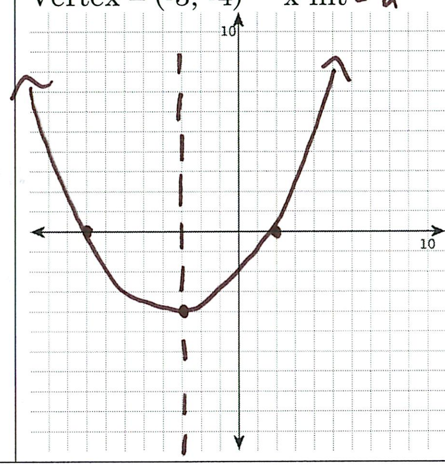
Vertex = (3, -1)   y-int = -6



Vertex = (-3, -6)   y-int = -4



Vertex = (-3, -4)   x-int = 2



= 2

Vertex = (3, 4)   x-int = 1