Graphing Parabolas

It is possible to find the vertex of a parabola quickly when the equation is written in a form such as $y - 5 = 3(x - 4)^2$. This form is similar to the point-slope form of the linear function equation, except that the (x – 4) is squared.

a) Transform the above quadratic function to the form $y = ax^2 + bx + c$.

$y - 5 = 3(x-4)(x-4)$

$y - 5 = 3(x^2 - 4x - 4x + 16)$

$y - 5 = 3(x^2 - 8x + 16)$

$y - 5 = 3x^2 - 24x + 48$

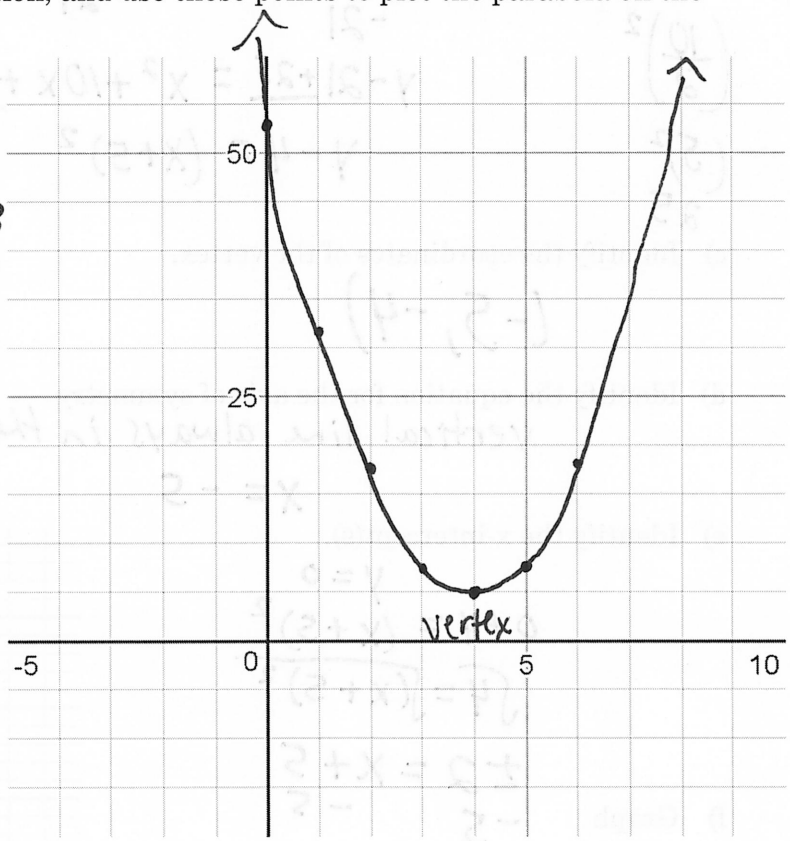
$y - 5 = 3x^2 - 24x + 48$

$+5 \qquad +5$

$y = 3x^2 - 24x + 53$

b) Complete the table for the above equation, and use these points to plot the parabola on the axes.

X	Y	
0	53	
1	32	$3(1)^2 - 24(1) + 53$
2	17	$3(2)^2 - 24(2) + 53$
3	8	$3(3)^2 - 24(3) + 53$
4	5	$3(4)^2 - 24(4) + 53$
5	8	$3(5)^2 - 24(5) + 53$
6	17	$3(6)^2 - 24(6) + 53$



c) Mark the vertex on the graph, and write down the ordered pair.

(4, 5)

d) Where do the coordinates of the vertex show up in the equation?

$y - 5 = 3(x-4)^2$

opp. of y-coord. ↑ opp. of x-coord.

e) How could you find the values of y for x = 7, and x = 8 without substituting these numbers into the equation?

use symmetry

From the previous activity, you saw that equations in the form $y - k = a(x - h)^2$ were easy to identify the coordinates of the vertex. This form is called vertex form. vertex (h, k)

We will now learn how to transform an equation from $y = ax^2 + bx + c$ form into vertex form. We will also learn how to graph it by finding the y-intercept, the vertex, the axis of symmetry, and the x-intercept(s).

Example: Consider the function $y = x^2 + 10x + 21$.

a) Identify the y-intercept. Let $x = 0$

$y = 0^2 + 10(0) + 21$ $\quad y = 21$ $\quad (0, 21)$

b) Transform the equation to vertex form by completing the square.

$y = x^2 + 10x + 21$

$-21 \qquad\qquad -21$

$y - 21 + 25 = x^2 + 10x + 25$

$y - 4 = (x+5)^2$

$\left(\frac{10}{2}\right)^2$

$(5)^2$

25

c) Identify the coordinates of the vertex.

$(-5, -4)$

d) Identify the equation for the axis of symmetry

vertical line always in the form $x = \#$ (x-coord. of vertex)

$x = -5$

e) Identify the x-intercept(s).

$y = 0$

$0 + 4 = (x+5)^2$

$\sqrt{4} = \sqrt{(x+5)^2}$

$\pm 2 = x + 5$

$-5 \qquad -5$

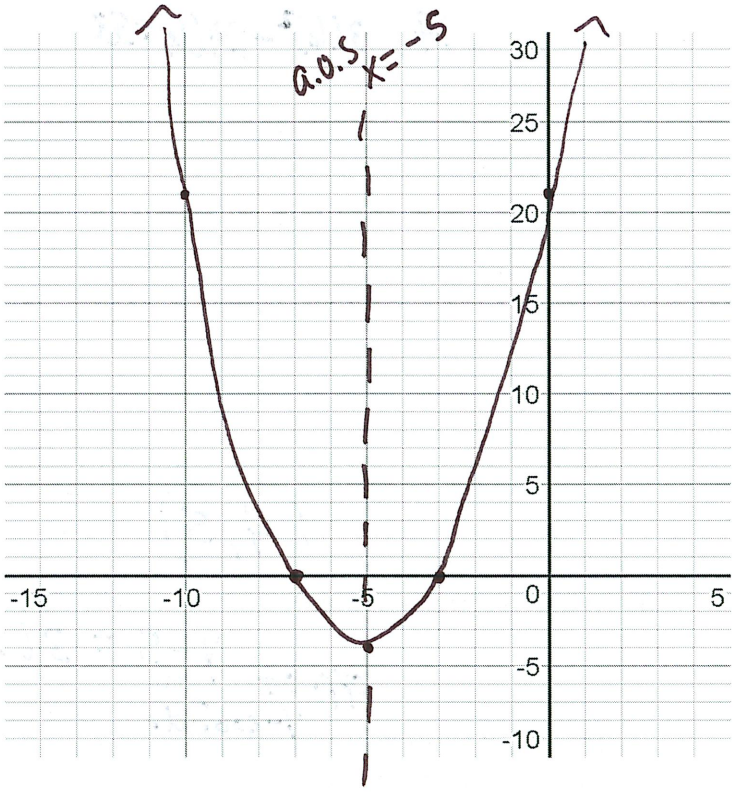
$-5 \pm 2 = x$

$-5 + 2 = -3$

$-5 - 2 = -7$

$(-3, 0)$, $(-7, 0)$

f) Graph



Break for Practice: Use the method of completing the square to find each of the requested pieces of information, and graph the parabola.

$y = x^2 + 6x + 13$

$-13 \quad -13$

$y - 13 = x^2 + 6x$

$\left(\frac{6}{2}\right)^2 = (3^2) = 9$

$y - 13 + 9 = x^2 + 6x + 9$

$y - 4 = (x+3)^2$

$y = 0$

$\sqrt{-4} = \sqrt{(x+3)^2}$

$\pm 2i = x + 3$

$-3 \quad -3$

$-3 \pm 2i = x$

imaginary

y-intercept

(0,13)

Vertex

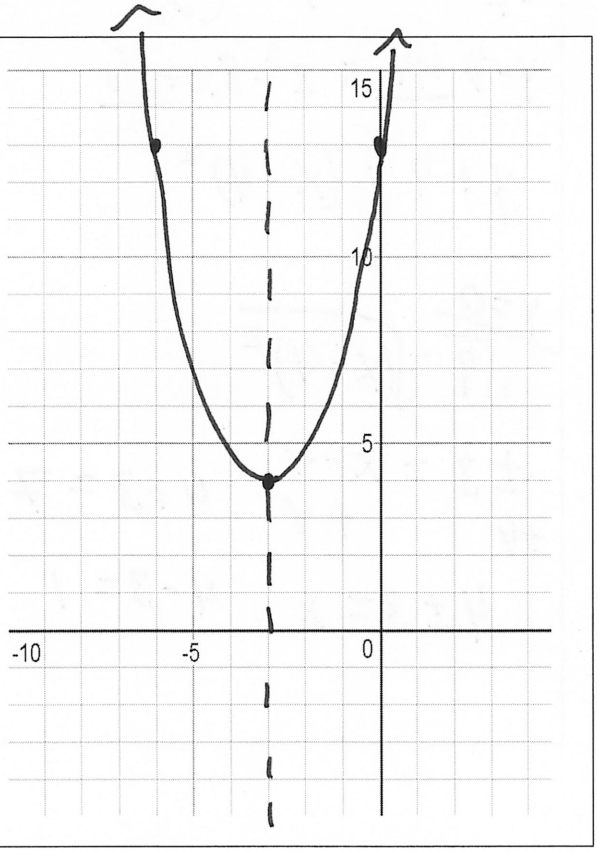
(-3, 4)

Axis

$x = -3$

x-intercept(s)

none



Extended Practice: Use the method of completing the square to find each of the requested pieces of information, and graph the parabola.

1. $y = x^2 - 6x + 8$

$-8 \quad -8$

$y - 8 \_\_\_ = x^2 - 6x \_\_\_$

$\left(\frac{-6}{2}\right)^2 = (-3)^2 = 9$

$y - 8 + 9 = x^2 - 6x + 9$

$y + 1 = (x-3)^2$

$y = 0$

$\sqrt{1} = \sqrt{(x-3)^2}$

$\pm 1 = x - 3$

$+3 \quad +3$

$3 \pm 1 = x$

$3 + 1 = 4$

$3 - 1 = 2$

y-intercept

(0,8)

Vertex

(3, -1)

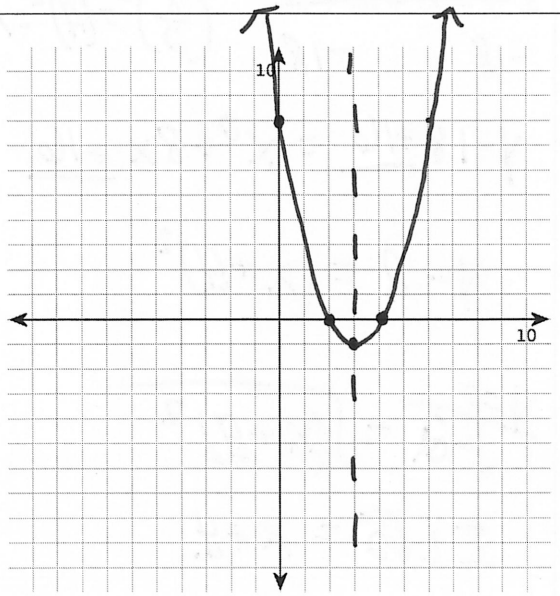
Axis

$x = 3$

x-intercept(s)

(4,0)

(2,0)



2. $y = x^2 - 8x + 7$

$-7 \qquad -7$ $\left(\frac{-8}{2}\right)^2 = (-4)^2 = 16$

$y - 7 \underline{+16} = x^2 - 8x \underline{+16}$

$y + 9 = (x-4)^2$

$y = 0$

$\sqrt{9} = \sqrt{(x-4)^2}$

$\pm 3 = x - 4$

$+4 \qquad +4$

$4 + 3 = 7$

$4 \pm 3 = x$

$4 - 3 = 1$

y-intercept

$(0, 7)$

Vertex

$(4, -9)$

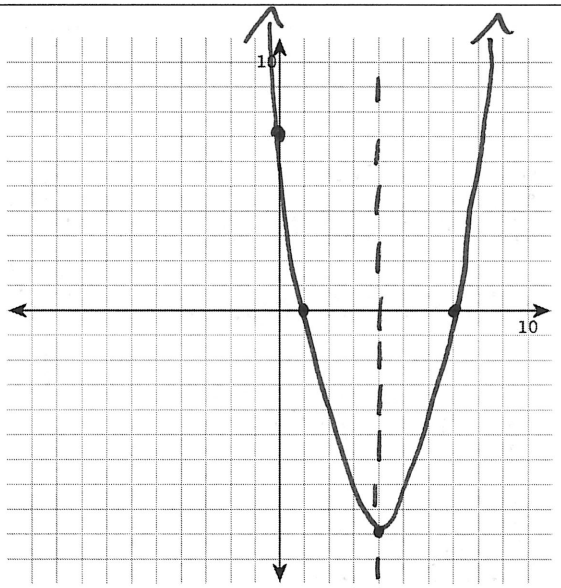
Axis

$x = 4$

x-intercept(s)

$(7, 0)$

$(1, 0)$



$y = x^2 + 4x + 8$

3. ~~$y = x^2 + 8x + 18$~~

$-8 \qquad -8$ $\left(\frac{4}{2}\right)^2 = (2)^2 = 4$

$y - 8 \underline{+4} = x^2 + 4x \underline{+4}$

$y - 4 = (x+2)^2$

$y = 0$

$\sqrt{-4} = \sqrt{(x+2)^2}$

$\pm i\sqrt{4} = x + 2$

imaginary

y-intercept

$(0, 8)$

Vertex

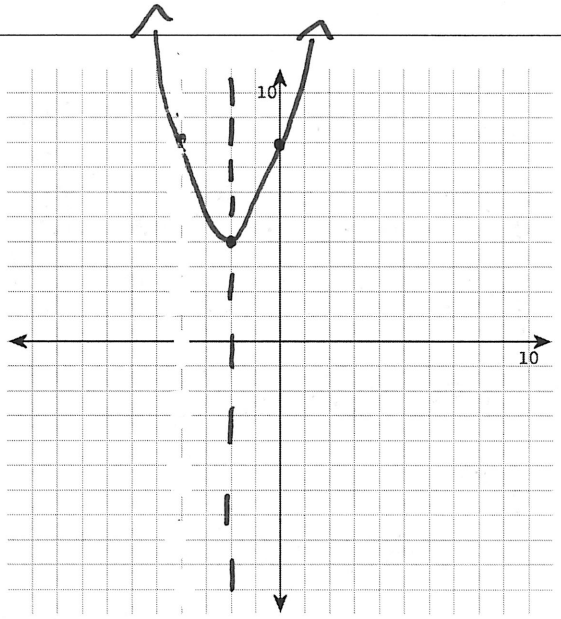
$(-2, 4)$

Axis

$x = -2$

x-intercept(s)

none



Next we should see what we need to adjust to work with equations that have a coefficient in front of the x^2

Break for Practice: Complete the following.

1. $y = 3x^2 + 30x + 72$

$-72 \quad -72 \quad \left(\frac{10}{2}\right)^2 = (5)^2 = 25$

$y - 72 = 3x^2 + 30x$

$y - 72 \underline{+75} = 3(x^2 + 10x \underline{+25})$

$y + 3 = 3(x+5)^2$

$y = 0$

$\frac{3}{3} = \frac{3(x+5)^2}{3}$

$\sqrt{1} = \sqrt{(x+5)^2}$

$\pm 1 = x + 5$

$-5 \quad -5$

$x = -5 \pm 1$

$-5+1 = -4$

$-5-1 = -6$

y-intercept

$(0, 72)$

Vertex

$(-5, -3)$

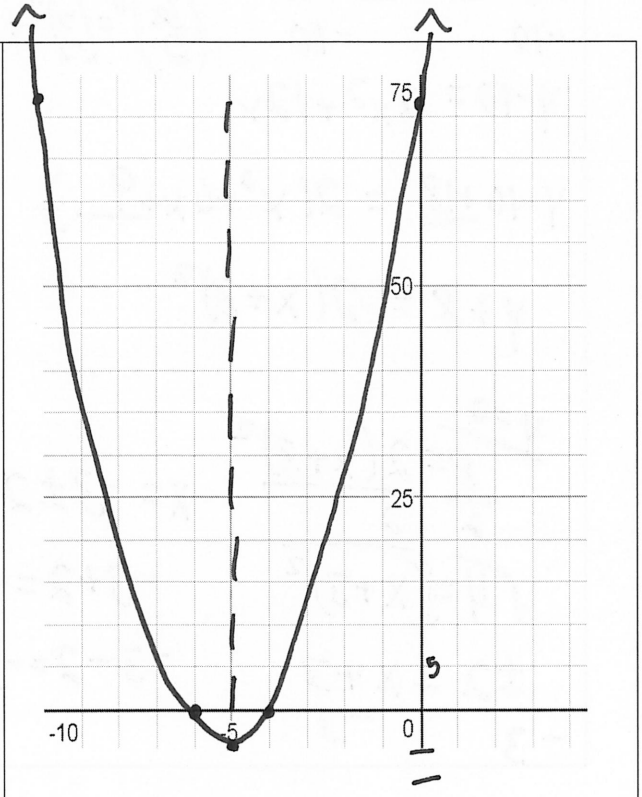
Axis

$x = -5$

x-intercept(s)

$(-4, 0)$

$(-6, 0)$



2. $y = -3x^2 + 12x - 42$

$+42 \quad +42 \quad \left(\frac{-4}{2}\right)^2 = (-2)^2 = 4$

$y + 42 = -3x^2 + 12x$

$y + 42 \underline{-12} = -3(x^2 - 4x \underline{+4})$

$y + 30 = -3(x-2)^2$

$y = 0$

$\frac{30}{-3} = \frac{-3(x-2)^2}{-3}$

$\sqrt{-10} = \sqrt{(x-2)^2}$

$\pm i\sqrt{10} = x - 2$

imaginary

y-intercept

$(0, -42)$

Vertex

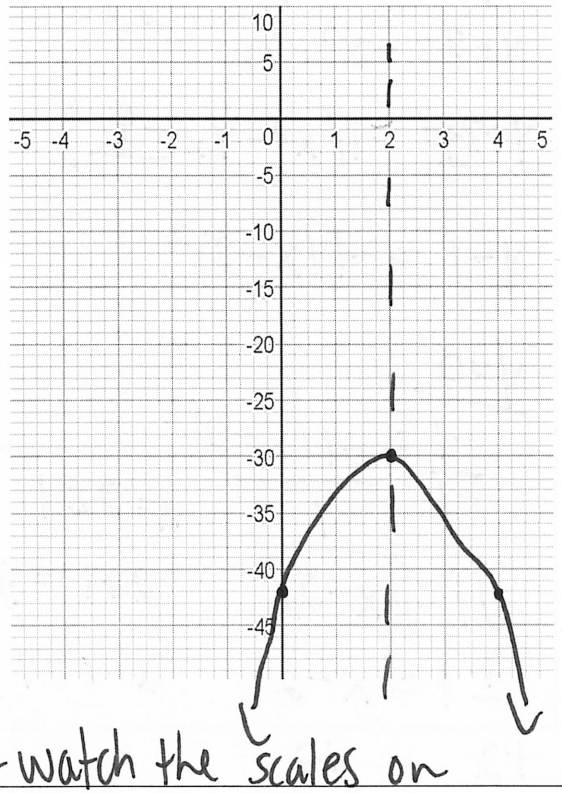
$(2, -30)$

Axis

$x = 2)$

x-intercept(s)

none



*watch the scales on the axes

Extended Practice: Complete the following.

1. $y = 2x^2 + 12x + 10$

$-10 \quad -10$

$y - 10 = 2x^2 + 12x$

$\left(\frac{6}{2}\right)^2 = (3)^2 = 9$

$y - 10 \underline{+18} = 2(x^2 + 6x \underline{+9})$

$y + 8 = 2(x+3)^2$

$y = 0$

$\frac{8}{2} = \frac{2(x+3)^2}{2}$

$\sqrt{4} = \sqrt{(x+3)^2}$

$\pm 2 = x + 3$

$-3 \quad -3$

$x = -3 \pm 2$

$-3 + 2 = -1$

$-3 - 2 = -5$

y-intercept

$(0, 10)$

Vertex

$(-3, -8)$

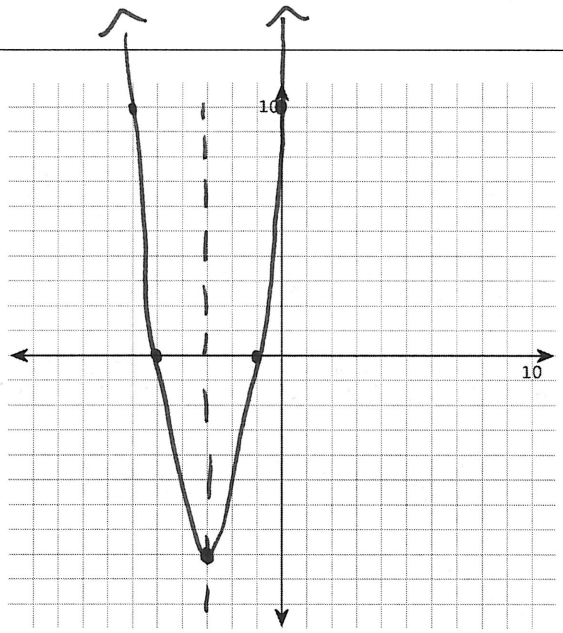
Axis

$x = -3$

x-intercept(s)

$(-1, 0)$

$(-5, 0)$



2. $y = -x^2 + 4x - 3$

$+3 \quad +3$

$y + 3 = -x^2 + 4x$

$\left(\frac{-4}{2}\right)^2 = (-2)^2 = 4$

$y + 3 \underline{+4} = -1(x^2 - 4x \underline{+4})$

$y + 1 = -1(x-2)^2$

$y = 0$

$\frac{-1}{-1} = \frac{-1(x-2)^2}{-1}$

$\sqrt{1} = \sqrt{(x-2)^2}$

$\pm 1 = x - 2$

$+2 \quad +2$

$x = 2 \pm 1$

$2 + 1 = 3$

$2 - 1 = 1$

y-intercept

$(0, -3)$

Vertex

$(2, 1)$

Axis

$x = 2$

x-intercept(s)

$(3, 0)$

$(1, 0)$