Algebra II
Unit 7 Review

Name Answer Key

1. Solve the following equations ***using the method of completing the square***.

a) $x^2 + 6x - 8 = 0$

$\left(\frac{6}{2}\right)^2 = (3)^2 = 9$

$+8 \quad +8$

$x^2 + 6x + 9 = 8 + 9$

$\sqrt{(x+3)^2} = \sqrt{17}$

$x + 3 = \pm\sqrt{17}$

$-3 \quad -3$

$\boxed{x = -3 \pm \sqrt{17}}$

b) $x^2 = 10x - 30$

$\left(\frac{-10}{2}\right)^2 = (-5)^2 = 25$

$-10x \quad -10x$

$x^2 - 10x + 25 = -30 + 25$

$\sqrt{(x-5)^2} = \sqrt{-5}$

$x - 5 = \pm i\sqrt{5}$

$+5 \quad +5$

$\boxed{x = 5 \pm i\sqrt{5}}$

2. Solve the following equations using the ***quadratic formula***:

a) $3x^2 + x - 1 = 0$

$a = 3$
$b = 1$
$c = -1$

$x = \frac{-1 \pm \sqrt{1^2 - 4(3)(-1)}}{2(3)}$

$x = \frac{-1 \pm \sqrt{13}}{6}$

$\boxed{x = \frac{-1 \pm \sqrt{13}}{6}}$

b) $5x^2 = -3x + 2$

$+3x - 2 \quad +3x - 2$

$5x^2 + 3x - 2 = 0$

$a = 5$
$b = 3$
$c = -2$

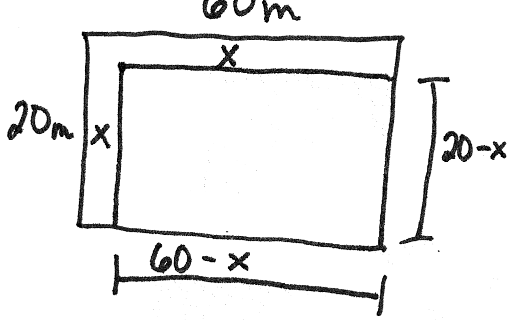
$x = \frac{-3 \pm \sqrt{3^2 - 4(5)(-2)}}{2(5)}$

$= \frac{-3 \pm \sqrt{49}}{10}$

$= \frac{-3 \pm 7}{10}$ → $\frac{-3+7}{10} = \frac{4}{10} = \frac{2}{5}$ → $\frac{-3-7}{10} = \frac{-10}{10} = -1$

$\boxed{x = -1, \frac{2}{5}}$

3. A rectangular corner lot has dimensions 20 meters by 60 meters. When the two adjoining streets are each widened by the same amount, one fourth of its area is lost. To the nearest meter, find the new dimensions of the lot.



$A_{orig} = 60(20) = 1200$

$A_{new} = 1200 - \frac{1}{4}\left(\frac{1200}{1}\right)$

$A_{new} = 900\text{ m}^2$

$900 = (20 - x)(60 - x)$

$900 = 1200 - 20x - 60x + x^2$

$-900 \quad -900$

$0 = x^2 - 80x + 300$

$a = 1$
$b = -80$
$c = 300$

$x = \frac{80 \pm \sqrt{(-80)^2 - 4(1)(300)}}{2(1)}$

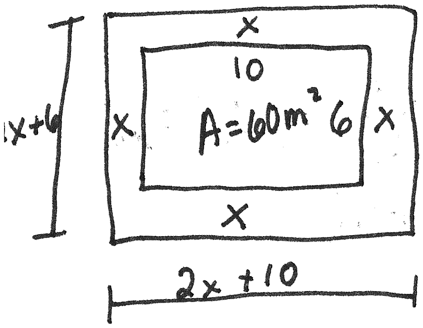
$x = \frac{80 \pm \sqrt{5200}}{2}$

$x = \frac{80 \pm 72.11}{2}$ → $\frac{80 + 72.11}{2} \approx 76$ too big → $\frac{80 - 72.11}{2} \approx 3.95 \approx 4$

$60 - 4 = 56$
$20 - 4 = 16$

$\boxed{16\text{m} \times 56\text{m}}$

4. A swimming pool 6 m wide and 10 m long is to be surrounded by a walk of uniform width. The area of the walk happens to equal the area of the pool. What is the width of the walk? $a = 4, b = 36 \quad c = -60$



$A_{walkway} = 60m^2$

$A_{walk} = A_{large} - A_{pool}$

$60 = (2x+6)(2x+10) - 60$

$60 = 4x^2 + 20x + 12x + \cancel{60} - \cancel{60}$

$-60 \qquad\qquad\qquad -60$

$0 = 4x^2 + 32x - 60$

$x = \dfrac{-32 \pm \sqrt{32^2 - 4(4)(-60)}}{2(4)}$

$= \dfrac{-32 \pm \sqrt{2256}}{8}$

$= \dfrac{-32 + 44.54}{8} = \boxed{1.57m}$

$= \dfrac{-32 - 44.54}{8} = -9.\cancel{87}$

5. Find the type of solutions for each equation by calculating the ***discriminant***. **You must show work to support your answer.**

a) $3x^2 - 5x + 6 = 0$

a b c

$D = (-5)^2 - 4(3)(6)$

$25 - 72$

D = -47 type of solutions 2 imag.

b) $9x^2 + 6x + 1 = 0$

a b c

$D = 6^2 - 4(9)(1)$

$D = 36 - 36$

$D = 0$

D = 0 type of solutions One 1 real (double) rational solutions

6. Find the value(s) of k for which the equation $3x^2 - 6x - k = 0$ has the following:

a b c

a) One real double root.

$0 = (-6)^2 - 4(3)(-k)$

$0 = 36 + 12k$

$-36 \quad -36$

$-36 = 12k$

$\boxed{k = -3}$

b) Two real roots.

$(-6)^2 - 4(3)(-k) > 0$

$36 + 12k > 0$

$-36 \qquad\quad -36$

$\dfrac{12k}{12} > \dfrac{-36}{12}$

$\boxed{k > -3}$

c) Two imaginary roots.

$(-6)^2 - 4(3)(-k) < 0$

$36 + 12k < 0$

$-36 \qquad\quad -36$

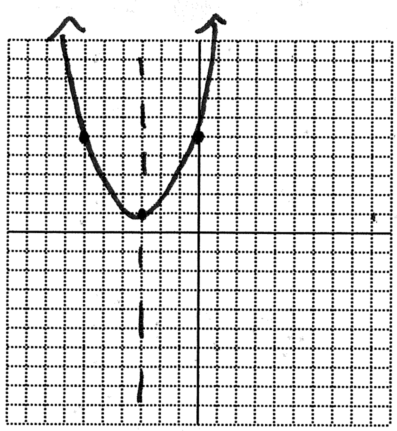
$\dfrac{12k}{12} < \dfrac{-36}{12}$

$\boxed{k < -3}$

7. Sketch a graph of each parabola described. The axis of symmetry for each parabola is vertical.

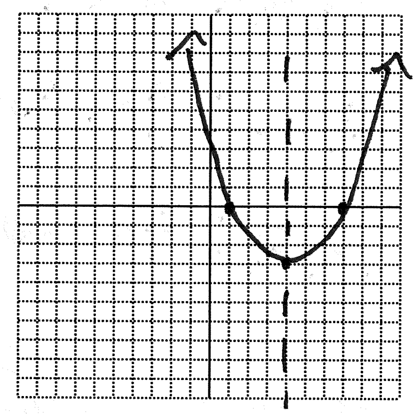
a) Vertex = (-3, 1)

y-int = 5



b) Vertex = (4, -3)

x-int = 1



8. Consider the quadratic function y = 4x² – 8x + 7.

a) What is the y-intercept?

$(0, 7)$

b) Transform the equation into vertex form.

$$y - 7 = 4x^2 - 8x$$

$$\left(\frac{-2}{2}\right)^2 = (-1)^2 = 1$$

$$y - 7 = 4(x^2 - 2x)$$

$$y - 7 \underline{+ 4} = 4(x^2 - 2x + 1)$$

$$\boxed{y - 3 = 4(x-1)^2}$$

9. An equation was put into vertex form and the result was y + 7 = 7(x – 3)².

a) State the coordinates of the vertex.

$(3, -7)$

b) State the equation of the axis of symmetry.

$x = 3$

c) State the coordinates of the x-intercepts.

$y = 0$

$$\frac{7}{7} = \frac{7(x-3)^2}{7}$$

$$\sqrt{1} = \sqrt{(x-3)^2}$$

$$\pm 1 = x - 3$$

$$+3 \qquad +3$$

$$3 \pm 1 = x$$

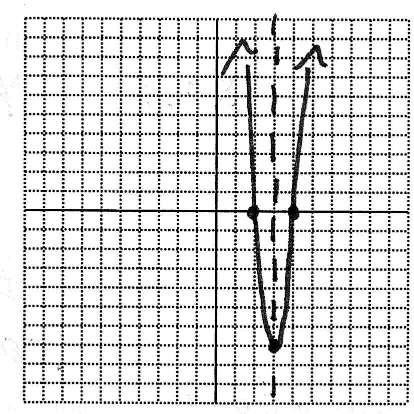
$(4, 0)$

$(2, 0)$

$3 + 1 = 4$

$3 - 1 = 2$

d) Graph the equation.



$ax^2+bx+c=y$

10. Find the equation for the parabola that contains the points (1, 6), (3, 26), and (-2, 21).

$(1,6)$: $a+b+c=6$

$(3,26)$: $9a+3b+c=26$

$(-2,21)$: $4a-2b+c=21$

$$\begin{array}{cc} A & B \\ \begin{bmatrix} 1 & 1 & 1 \\ 9 & 3 & 1 \\ 4 & -2 & 1 \end{bmatrix} & \begin{bmatrix} 6 \\ 26 \\ 21 \end{bmatrix} \end{array}$$

$$A^{-1}(B)=\begin{bmatrix} 3 \\ -2 \\ 5 \end{bmatrix}$$

$$\boxed{y=3x^2-2x+5}$$

11. A study compared the speed (x), in miles per hour, and the average fuel economy (y), in miles per gallon, for cars. Some results are shown in the following table.

Speed (x)	Fuel Economy (y)
20	25
40	30
60	28

a) Assume the fuel economy (y) is a quadratic function of the speed (x). Write the particular equation expressing fuel economy (y) in terms of speed (x).

$(20,25)$: $400a+20b+c=25$

$(40,30)$ $1600a+40b+c=30$

$(60,28)$ $3600a+60b+c=28$

$$\begin{array}{cc} A^{-1} & B \\ \begin{bmatrix} 400 & 20 & 1 \\ 1600 & 40 & 1 \\ 3600 & 60 & 1 \end{bmatrix} & \begin{bmatrix} 25 \\ 30 \\ 28 \end{bmatrix} \end{array} = \begin{bmatrix} -.00875 \\ .775 \\ 13 \end{bmatrix}$$

$$\boxed{y=-.00875x^2+.775x+13}$$

b) What does your equation predict is the fuel economy of a car travelling at 15 miles per hour?

$x=15$ $\quad y=-.00875(15)^2+.775(15)+13$

$$\boxed{y=22.656 \text{ miles/gal}}$$

c) Is it possible to get a fuel economy of 35 miles per gallon with this model? Show work to prove your answer.

$y=35$:

$$-.00875x^2+.775x+13=35$$
$$-35 \quad -35$$
$$\underbrace{-.00875}_{a}x^2+\underbrace{.775}_{b}x\underbrace{-22}_{c}=0$$

no imaginal

$$x=\frac{-.775\pm\sqrt{.775^2-4(-.00875)(-22)}}{2(-.00875)}=\frac{-.775\pm\sqrt{-.17}}{-0.0175}$$

not possible