2. If x is jointly proportional to y and the square root of z, and x = 20 when y = 5 and z = 9, find x when y = 6 and z = 25.

$x = ky\sqrt{z}$

$20 = k(5)\sqrt{9}$

$\frac{20}{15} = \frac{k(5)(3)}{15}$

$\frac{4}{3} = k$

$x = \frac{4}{\cancel{3}}\left(\frac{\cancel{6}^{2}}{1}\right)\sqrt{25}$

$x = 8 \cdot 5$

$\boxed{x = 40}$

3. The electrical resistance of a wire varies directly as its length and inversely as the square of its diameter. 100 m of wire with a diameter of 6 mm has a resistance of 12 ohms. Eighty meters of a second wire of the same material has a resistance of 15 ohms. Find the diameter of the second wire.

r = resistance

l = length

d = diameter

$r = \frac{k \cdot l}{d^2}$

l = 100

d = 6

r = 12

$36(12) = \left(\frac{k \cdot 100}{6^2}\right)36$

$\frac{432}{100} = \frac{100k}{100}$

$4.32 = k$

l = 80

d = ?

r = 15

$\frac{15}{1} = \frac{4.32(80)}{d^2}$

$\frac{15d^2}{15} = \frac{345.6}{15}$

$\sqrt{d^2} = \sqrt{23.04}$

$d \approx 4.8$

$\boxed{\text{diameter } 4.8\text{mm}}$

**Extended Practice**: Solve

1. If y varies inversely as x, and y = 3 when x = 6, find x when y = 18.

$y = \frac{k}{x}$

$6(3) = \frac{k}{6} \cdot 6$

$18 = k$

$\frac{18}{1} = \frac{18}{x}$

$\frac{18x}{18} = \frac{18}{18}$

$\boxed{x = 1}$

2. If z is inversely proportional to r, and z = 32 when r = 1.5, find r when z = 8.

$z = \frac{k}{r}$

$1.5(32) = \frac{k}{1.5} \cdot 1.5$

$48 = k$

$\frac{8}{1} = \frac{48}{r}$

$\frac{8r}{8} = \frac{48}{8}$

$\boxed{r = 6}$

3. If w is inversely proportional to the square of v, and w = 3 when v = 6, find w when v = 3.

$w = \frac{k}{v^2}$ $\quad 36(3) = \left(\frac{k}{6^2}\right)36$ $\quad w = \frac{108}{3^2} = \frac{108}{9}$

$108 = k$

$\boxed{w = 12}$

4. If p varies inversely as the square root of q, and p= 12 when q = 36, find p when q = 16.

$p = \frac{k}{\sqrt{q}}$ $\quad 6(12) = \frac{k}{\sqrt{36}} \cdot 6$ $\quad p = \frac{72}{\sqrt{16}} = \frac{72}{4}$

$72 = k$

$\boxed{p = 18}$

5. If z is jointly proportional to x and y, and z = 18 when x = 0.4 and y = 3, find z when x = 1.2 and y = 2.

$z = kxy$ $\quad 18 = k(0.4)(3)$ $\quad z = 15(1.2)(2)$

$\frac{18}{1.2} = \frac{1.2k}{1.2}$ $\quad \boxed{z = 36}$

$15 = k$

6. If w is jointly proportional to u and v, and w = 24 when u = 0.8 and v = 5, find u when w = 18 and v = 2.

$w = k \cdot u \cdot v$ $\quad \frac{24}{4} = \frac{k(0.8)(5)}{4}$ $\quad \frac{18}{12} = \frac{6u(2)}{12}$

$6 = k$ $\quad \boxed{\frac{3}{2} = u}$

7. If s varies directly as r and inversely as t, and s = 10 when r = 5 and t = 3, for what value of t will s = 3 when r = 4?

$s = \frac{k \cdot r}{t}$ $\quad 3(10) = \left(\frac{k(5)}{3}\right)3$ $\quad \frac{3}{1} = \frac{6(4)}{t}$

$\frac{30}{5} = \frac{5k}{5}$ $\quad \frac{3t}{3} = \frac{24}{3}$

$6 = k$ $\quad \boxed{t = 8}$

8. Suppose that r varies directly as p and inversely as $q^2$, and that r = 27 when p = 3 and q = 2. Find r when p= 2 and q = 3.

$$r = \frac{k \cdot p}{q^2} \quad 4(27) = \left(\frac{k(3)}{2^2}\right)4 \quad r = \frac{36(2)}{3^2}$$

$$\frac{108}{3} = \frac{3k}{3} \qquad r = \frac{72}{9}$$

$$36 = k \qquad \boxed{r = 8}$$

9. The frequency of a radio signal varies inversely as the wave length. A signal of frequency 1200kilohertz (kHz), which might be the frequency of an AM radio station, has wave length 250 m. What frequency has a signal of wave length 400m?

f = frequency

l = wave length

$$f = \frac{k}{l} \quad 250(1200) = \left(\frac{k}{250}\right)250$$

$$300{,}000 = k$$

$$f = \frac{300{,}000}{400}$$

$$\boxed{f = 750 \text{ kHz}}$$

10. The stretch in a wire under a given tension varies directly as the length of the wire and inversely as the square of its diameter. A wire having length 2 m and diameter 1.5 mm stretches 1.2 mm. If a second wire of the same material (and under the same tension) has length 3 m and diameter 2.0 mm, find the amount of stretch.

W = wire stretch

l = length

d = diameter

$$w = \frac{k \cdot l}{d^2} \qquad l = 2,\ d = 1.5,\ w = 1.2 \qquad 1.2 = \frac{k(2)}{1.5^2}$$

$$2.25(1.2) = \left(\frac{2k}{2.25}\right)2.25$$

$$\frac{2.7}{2} = \frac{2k}{2}$$

$$1.35 = k$$

$$l = 3,\ d = 2.0,\ w = ? \qquad w = \frac{1.35(3)}{2^2}$$

$$\boxed{w = 1.01 \text{ mm}}$$

# Linear Interpolation

We will finish this unit with the topic of linear interpolation. Linear interpolation is a method to estimate values that are not given in a table or chart. It makes use of ratios and proportions.

**Break for Practice**:

1. Use linear interpolation and the table to find these values to the nearest integer.

   a) Find the approximate number of bachelor's degrees that were awarded in computer science in 1971?

| Year | Bachelor's degrees awarded in computer science |
|---|---|
| 1968 | 459 |
| 1972 | 3,402 |
| 1976 | 5,700 |
| 1980 | 11,154 |
| 1984 | 32,172 |

Steps

1. identify what gap/#'s/data the desired goal goes between (2 values)
2. rewrite info with desired in between
3. calculate difference of gap between 1st + 2nd, 1st + 3rd
4. Set up proportion and solve
5. Add solution to the # before desired or subtract

   b) Find the approximate year that 10,000 bachelor's degrees were awarded in computer science.

b)

| | | |
|---|---|---|
| 1976 | 5700 | |
| x | 10,000 | 4300 |
| 1980 | 11,154 | 5454 |

$d$ (1976 to x), 4 (1976 to 1980)

$$\frac{d}{4} = \frac{4300}{5454}$$

$$\frac{5454d}{5454} = \frac{17200}{5454}$$

$$d \approx 3.15 \rightarrow 3 \text{ yrs.}$$

$$x = 1976 + 3$$

$$\boxed{x = 1979}$$

a)

| | | |
|---|---|---|
| 1968 | 459 | |
| 1971 | x | d |
| 1972 | 3,402 | 2943 |

3 (1968 to 1971), 4 (1968 to 1972)

$$\frac{3}{4} = \frac{d}{2943}$$

$$\frac{4d}{4} = \frac{8829}{4}$$

$$d = 2207.25$$

$$x = 2207.25 + 459 = 2666.25$$

$$\boxed{x = 2{,}666 \text{ degrees in } 1971}$$

2. Use linear interpolation and the table to find these values to the nearest integer.

The table gives the temperature in degrees Fahrenheit on a spring day in Boston, MA.

| Time (pm) | Temperature (F) |
|---|---|
| 1:00 | 68 |
| 2:00 | 66 |
| 3:00 | 63 |
| 4:00 | 59 |
| 5:00 | 53 |
| 6:00 | 45 |
| 7:00 | 39 |

a) Approximate the temperature at 3:40 pm.

$$60\left[40\left[\begin{array}{ll} 3{:}00 & 63 \\ 3{:}40 & x \end{array}\right] d \\ \begin{array}{ll} 4{:}00 & 59 \end{array}\right] 4$$

$$\frac{40}{60} = \frac{d}{4}$$

$$\frac{160}{60} = \frac{60d}{60}$$

$$2.67 = d$$

$$x = 63 - 2.67$$

$$x = 60.33$$

$$\boxed{60°F}$$

b) At about what time was the temperature 40°?

$$60\left[d\left[\begin{array}{ll} 6{:}00 & 45 \\ x & 40 \end{array}\right] 5 \\ \begin{array}{ll} 7{:}00 & 39 \end{array}\right] 6$$

$$\frac{d}{60} = \frac{5}{6}$$

$$\frac{6d}{6} = \frac{300}{6}$$

$$d = 50 \text{ min}$$

$$x = 6{:}00 + 50 \text{ min}$$

$$\boxed{6{:}50 \text{ p.m}}$$

**Extended Practice**: Solve each problem using linear interpolation.

1. Consider the table of population figures for the following questions.

| Year | U.S. Population in millions |
|---|---|
| 1900 | 76 |
| 1910 | 92 |
| 1920 | 106 |
| 1930 | 123 |
| 1940 | 132 |
| 1950 | 151 |
| 1960 | 179 |
| 1970 | 203 |
| 1980 | 227 |
| 1990 | 243 |

a) Approximate the population in 1915.

$$10\left[5\left[\begin{array}{c}1910\\1915\end{array}\right.\begin{array}{c}\\1920\end{array}\right.\qquad \left.\left.\begin{array}{c}92\\x\end{array}\right]d\ \begin{array}{c}\\106\end{array}\right]14$$

$$\frac{5}{10}=\frac{d}{14} \qquad d=7$$

$$x=92+7$$

$$\frac{70}{10}=\frac{10d}{10} \qquad \boxed{99 \text{ million}}$$

b) Approximate the population in 1963.

$$10\left[3\left[\begin{array}{c}1960\\1963\end{array}\right.\begin{array}{c}\\1970\end{array}\right.\qquad \left.\left.\begin{array}{c}179\\x\end{array}\right]d\ \begin{array}{c}\\203\end{array}\right]24$$

$$\frac{3}{10}=\frac{d}{24} \qquad d=7.2$$

$$\frac{72}{10}=\frac{10d}{10} \qquad x=179+7.2\approx\boxed{186.2 \text{ million}}$$

c) Approximate the year that the population was 100 million.

$$10\left[d\left[\begin{array}{c}1910\\x\end{array}\right.\begin{array}{c}\\1920\end{array}\right.\qquad \left.\left.\begin{array}{c}92\\100\end{array}\right]8\ \begin{array}{c}\\106\end{array}\right]14$$

$$x=1910+5.71=1915.71$$

$$\boxed{1916}$$

$$\frac{d}{10}=\frac{8}{14} \qquad \frac{14d}{14}=\frac{80}{14} \qquad d=5.71$$

d) Approximate the year that the population was 200 million.

$$10\left[d\left[\begin{array}{c}1960\\x\end{array}\right.\begin{array}{c}\\1970\end{array}\right.\qquad \left.\left.\begin{array}{c}179\\200\end{array}\right]21\ \begin{array}{c}\\203\end{array}\right]24$$

$$x=1960+8.75=1968.75$$

$$x=\boxed{1969}$$

$$\frac{d}{10}=\frac{21}{24}$$

$$\frac{24d}{24}=\frac{210}{24} \qquad d=8.75$$

2. The table gives the density of dry air at various altitudes.

| Altitude (m) | 0 | 500 | 1000 | 1500 | 2000 | 2500 | 3000 | 3500 |
|---|---|---|---|---|---|---|---|---|
| Density (kg/m³) | 1.225 | 1.167 | 1.112 | 1.058 | 1.007 | 0.957 | 0.909 | 0.863 |

a) Approximate the density at an altitude of 1200 m.

500 [ 200 [ 1000 / 1200 / 1500     1.112 / x / 1.058     ] d ] 0.054

$x = 1.112 - .0216$

$x = \boxed{1.090 \text{ kg/m}^3}$

$\frac{2\cancel{00}}{5\cancel{00}}$

$\frac{2}{5} = \frac{d}{.054}$

$\frac{.108}{5} = \frac{5d}{5}$     $d = .0216$

b) Approximate the density at an altitude of 3200 m.

500 [ 200 [ 3000 / 3200 / 3500     0.909 / x / 0.863     ] d ] .046

$x = .909 - .0184$

$x = \boxed{.891 \text{ kg/m}^3}$

$\frac{2\cancel{00}}{5\cancel{00}}$

$\frac{2}{5} = \frac{d}{.046}$     $d = .0184$

$\frac{.092}{5} = \frac{5d}{5}$

c) Approximate the altitude for dry air with a density of 1.200 kg/m³.

500 [ d [ 0 / x / 500     1.225 / 1.2 / 1.167     ] .025 ] .058

$x = 0 + 215.517$

$x = \boxed{216 \text{ m}}$

$\frac{d}{500} = \frac{.025}{.058}$     $\frac{.058d}{.058} = \frac{12.5}{.058}$

$d = 215.517$

d) Approximate the altitude for dry air with a density of 0.930 kg/m³.

500 [ d [ 2500 / x / 3000     0.957 / 0.930 / 0.909     ] .027 ] .048

$x = 2500 + 281.25$

$= 2781.25$

$x = \boxed{2781 \text{ m}}$

$\frac{d}{500} = \frac{.027}{.048}$     $d = 281.25$

$\frac{.048d}{.048} = \frac{13.5}{.048}$