Break for Practice: Solve each problem

1. $5.1^x = 134.2$

$\log_{5.1} 134.2 = x$

$\frac{\log 134.2}{\log 5.1} = x$

$\boxed{3.007 = x}$

2. $15.7^x = 0.093$

$\log_{15.7} 0.093 = x$

$\frac{\log .093}{\log 15.7} = x$

$\boxed{-.863 = x}$

3. $e^x = 47.2$

$\log_e 47.2 = x$ $\quad$ * $\log_e = \ln$

$\ln 47.2 = x$

$\boxed{3.854 = x}$

4. $10^x = 170$

$\underline{\log_{10}} 170 = x$

calculator

$2.2304 = x$

5. $x = 10^{5.7}$

$x = 501187.2336$

6. $x = e^1$

$x = 2^{nd}$ $\boxed{\ln}$ 1 $\boxed{\text{enter}}$

e^x

$x = 2.718$

7. $x = e^{4.1}$

$x = 60.3403$

8. $\log 3x = 1.2$

$\log_{10} 3x = 1.2$

$\frac{10^{1.2}}{3} = \frac{3x}{3}$

$\boxed{5.283 = x}$

9. $\ln x = 5.3$

$\ln_e x = 5.3$

$e^{5.3} = x$

$\boxed{200.3368 = x}$

10. $\log 5^x = 3$

$\frac{x \cdot \cancel{\log 5}}{\cancel{\log 5}} = \frac{3}{\log 5}$

$\boxed{x = 4.292}$

Extended Practice: Solve each problem

1. $10^x = 211$

$\log_{10} 211 = x$

$\boxed{2.3243 = x}$

2. $e^x = 28$

$\log_e 28 = x$

$\ln 28 = x$

$\boxed{3.3322 = x}$

3. $2.7^x = 88$

$\log_{2.7} 88 = x$

$\frac{\log 88}{\log 2.7} = x$

$\boxed{4.5078 = x}$

4. $x = 10^{3.4}$

$x = 2511.8864$

5. $x = e^{4.2}$

$\boxed{x = 66.6863}$

6. $\log 4x = 1.9$

$\log_{10} 4x = 1.9$

$\frac{10^{1.9}}{4} = \frac{4x}{4}$

$\boxed{19.8582 = x}$

7. $\ln 7x = 3.8$

$\ln_e 7x = 3.8$

$\frac{e^{3.8}}{7} = \frac{7x}{7}$

$\boxed{6.3859 = x}$

8. $log_5 4x = 3.1$

$\frac{5^{3.1}}{4} = \frac{4x}{4}$

$\boxed{36.7068 = x}$

9. $log_6 2x = -1.2$

$\frac{6^{-1.2}}{2} = \frac{2x}{2}$

$\boxed{0.0582 = x}$

10. $\log 3.1^x = 1.8$

$\frac{x \cdot \cancel{\log 3.1}}{\cancel{\log 3.1}} = \frac{1.8}{\log 3.1}$

$\boxed{x = 3.6633}$

11. $\ln 2.7^x = 8$

$\frac{x \cdot \cancel{\ln 2.7}}{\cancel{\ln 2.7}} = \frac{8}{\ln 2.7}$

$\boxed{x = 8.0544}$

12. $log_7 x = 5$

$7^5 = x$

$\boxed{16807 = x}$

Finding Exponential Functions

Before we look at applications of exponential and logarithmic functions, we need to learn how to find the equations when given two points.

Remember: Exponential functions are in the form $f(x) = a \cdot b^x$. Since there are two unknowns (a and b), we will need two points to find the correct equation.

Break for Practice:

1. If f(x) is exponential and f(1) = 10 and f(3) = 250, find the formula.

x x

1 (1, 10)

2 (3, 250)

$f(x) = a \cdot b^x$

$y = a \cdot b^x$

Pt 1) $10 = a \cdot b^1$

Pt 2) $250 = a \cdot b^3$

$\frac{250 = \cancel{a} \cdot b^3}{10 = \cancel{a} \cdot b^1}$

$\sqrt{25} = \sqrt{b^2}$

$5 = b$

$\frac{10}{5} = \frac{a \cdot 5^1}{5}$

$2 = a$

$\boxed{f(x) = 2 \cdot 5^x}$

* can't simply because exponents first

Steps

1. Write points
2. Create 2 equations $y = a \cdot b^x$
3. divide equations with larger "x" value on top
4. Solve for b
5. plug back into equation and solve for a
6. Write equation with a, b

a) Find f(2).

$f(2) = 2 \cdot 5^2$

$= 2 \cdot 25$

$\boxed{f(2) = 50}$

b) Find x if f(x) = 6250.

$\frac{6250}{2} = \frac{2 \cdot 5^x}{2}$

$3125 = 5^x$

$\log_5 3125 = x$

$\frac{\log 3125}{\log 5} = x$

$\boxed{5 = x}$

2. Find the formula for f(x) if f(8) = 40 and f(3) = 1280.

$(8,40)$

$(3,1280)$

$$\frac{40=\cancel{a}\cdot b^8}{1280=\cancel{a}\cdot b^3}$$

$$\sqrt[5]{\frac{1}{32}}=\sqrt[5]{b^5}$$

$$\frac{1}{2}=b$$

$$1280=a\left(\tfrac{1}{2}\right)^3$$

$$8\cdot 1280=a\left(\tfrac{1}{8}\right)\cdot 8$$

$$10240=a$$

$$\boxed{f(x)=10240\cdot\tfrac{1}{2}^{x}}$$

a) Find f(-2). $f(-2)=10240\left(\tfrac{1}{2}\right)^{-2}$

$$\boxed{f(-2)=40960}$$

b) Find x if f(x) = 163,840.

$$\frac{163{,}840}{10240}=\frac{10240\cdot\tfrac{1}{2}^{x}}{10240}$$

$$16=\tfrac{1}{2}^{x}$$

$$\log_{1/2}16=x$$

$$\frac{\log 16}{\log 1/2}=x$$

$$\boxed{-4=x}$$

Extended Practice:

1. Find the formula for f(x) if f(1) = 6 and f(4) = 162.

$(1,6)$

$(4,162)$

$$6=a\cdot b^1$$

$$162=a\cdot b^4$$

$$\frac{162=a\cdot b^4}{6=ab^1}$$

$$\sqrt[3]{27}=\sqrt[3]{b^3}$$

$$3=b$$

$$\frac{6}{3}=\frac{a\cdot 3^1}{3}$$

$$2=a$$

$$\boxed{f(x)=2\cdot 3^x}$$

a) Find f(8). $f(8)=2\cdot 3^8$

$$\boxed{f(8)=13122}$$

b) Find x if f(x) = 1,458

$$\frac{1458}{2}=\frac{2\cdot 3^x}{2}$$

$$729=3^x$$

$$\log_3 729=x$$

$$\frac{\log 729}{\log 3}=x$$

$$\boxed{6=x}$$

2. Find the formula for f(x) if f(2) = 40 and f(5) = 320.

(5, 320)
(2, 40)

$$\frac{320 = \cancel{a}\cdot b^5}{40 = \cancel{a}\cdot b^2}$$

$$\sqrt[3]{8} = \sqrt[3]{b^3}$$

$$2 = b$$

$$\frac{40}{4} = \frac{a\cdot 2^2}{4}$$

$$10 = a$$

$$\boxed{f(x) = 10\cdot 2^x}$$

c) Find f(-3).

$$f(-3) = 10\cdot 2^{-3}$$

$$f(-3) = \frac{10}{8} = \boxed{\frac{5}{4}}$$

d) Find x if f(x) = 0.15625

$$\frac{0.15625}{10} = \frac{10\cdot 2^x}{10}$$

$$.015625 = 2^x$$

$$\log_2 0.015625 = x \rightarrow \frac{\log .015625}{\log 2} = x$$

$$\boxed{-6 = x}$$

3. Find the formula for f(x) if f(2) = 1 and f(4) = 4.

(4, 4)
(2, 1)

$$\frac{4 = \cancel{a}\cdot b^4}{1 = \cancel{a}\cdot b^2}$$

$$\sqrt{4} = \sqrt{b^2}$$

$$2 = b$$

$$\frac{1}{4} = \frac{a\cdot 2^2}{4}$$

$$\frac{1}{4} = a$$

$$\boxed{f(x) = \frac{1}{4}\cdot 2^x}$$

e) Find f(12).

$$f(12) = \frac{1}{4}\cdot 2^{12}$$

$$= \frac{1}{4}(4096$$

$$\boxed{f(12) = 1024}$$

f) Find x if f(x) = 0.00390625

$$4(.00390625) = \left(\frac{1}{4}\cdot 2^x\right)4$$

$$.015625 = 2^x$$

$$\log_2 0.015625 = x$$

$$\frac{\log .015625}{\log 2} = x$$

$$\boxed{x = -6}$$

Exponential and Logarithmic Functions

Exponential and logarithmic functions have many applications. We will look at several.

Break for Practice: Assume that the population of the United States is increasing exponentially with time. The 1970 census showed that the population was about 203 million. The 1980 census showed that the population had grown to about 226 million.

a) Find the particular equation expressing population in terms of the number of years that have elapsed since 1970.

b) Use the formula to predict the population in 1990, 2000, and 2010.

c) Use the formula to predict the population this year.

d) Predict the year in which the population will reach 400 million.

e) According to the formula, what was the population when the Declaration of Independence was signed? The actual population was about 4 million. Why do you think our model gives a poor result in this case?

Extended Practice:

1. When rabbits were first brought to Australia last century, they had no natural enemies so their numbers increased rapidly. (busy bunnies!) Assume that there were 60,000 rabbits in 1865, and that by 1867 the number had increased to 2,400,000. Assume that the number of rabbits increased exponentially with the number of years that elapsed since 1865.

   a) Write the particular equation for this function.
   b) How many rabbits would you predict in 1870?
   c) According to the formula, when was the first pair of rabbits introduced into Australia?

2. Oliver Sudden is driving along a straight, level highway at 64 km/hr when his car runs out of gas. As he slows down, his speed decreases exponentially with the number of seconds since he ran out of gas, dropping to 48 km/hr after 10 seconds.

   a) Write the particular equation expressing speed in terms of time.
   b) Predict Oliver's speed after 25 seconds.
   c) At what time will Oliver's speed be 10 km/hr?

3. The pressure of the air in the Earth's atmosphere decreases exponentially with altitude above the surface of the Earth. The pressure at the Earth's surface (sea level) is about 14.7 pounds per square inch (psi) and the pressure at 2000 feet is approximately 13.5 psi.

   a) Write the particular equation expressing pressure in terms of altitude.
   b) Predict the pressure at Mount Everest (altitude 29,000 feet).
   c) Human blood at body temperature will boil if the pressure is below 0.9 psi. At what altitude would your blood start to boil if you were in an unpressurized airplane?

4. You accidentally inhale some mildly poisonous fumes. Twenty hours later you see a doctor. From a blood sample, she measures a poison concentration of 0.00372 milligrams per cubic centimeter (mg/cc), and tells you to come back in 8 hours. On the second visit, she measures a concentration of 0.00219 mg/cc. Let t be the number of hours that have elapsed since your first visit to the doctor, and let C be the concentration of poison in your blood, in mg/cc. Assume that C varies exponentially with t.

   a) Write the particular equation for this function.
   b) The doctor says you might have had serious body damage if the poison concentration was ever as high as 0.015 mg/cc. Based on your formula, was the concentration ever that high?
   c) You can resume normal activities when the poison concentration has dropped to 0.00010 mg/cc. How long after you inhaled the fumes will you be able to resume normal activities?