We can now use what we know about exponents to solve some exponential equations.

Example: Solve $3^x = 81$

$$3^x = 3^4$$

$$x = 4$$

81
$9 \quad 9$
$3\ 3 \quad 3\ 3$

Steps:

1. Write each side with the same base.
2. Set the exponents equal and solve.

Break for Practice: Solve each equation

1. $27^x = \frac{1}{9}$

$$3^{3x} = \frac{1}{3^2}$$

$$3^{3x} = 3^{-2}$$

$$\frac{3x}{3} = \frac{-2}{3}$$

$$\boxed{x = \frac{-2}{3}}$$

2. $7^{4-x} = 49^{x-1}$

$$7^{(4-x)} = 7^{2(x-1)}$$

$$4 - \cancel{x} = 2x - \cancel{2}$$

$$+2 \ +\cancel{x} \quad +x \ \cancel{+2}$$

$$\frac{6}{3} = \frac{3x}{3}$$

$$\boxed{2 = x}$$

3. $125^{x-3} = 5\sqrt{5}$

$$5^{3(x-3)} = 5^1 \cdot 5^{\frac{1}{2}}$$

$$5^{3x-9} = 5^{3/2}$$

$$2(3x - 9 = 3/2)$$

$$6x - 1\cancel{8} = 3$$

$$+1\cancel{8} \quad +18$$

$$\frac{6x}{6} = \frac{21}{6}$$

$$\boxed{x = \frac{7}{2}} \ \boxed{\text{or } 3.5}$$

4. $64^{x-1} = 4$

$$4^{3(x-1)} = 4^1$$

$$3x - \cancel{3} = 1$$

$$+\cancel{3} \quad +3$$

$$\frac{3x}{3} = \frac{4}{3}$$

$$\boxed{x = \frac{4}{3}}$$

Extended Practice: Solve

1. $3^x = \frac{1}{27} = \frac{1}{3^3}$

$3^x = 3^{-3}$

$\boxed{x = -3}$

2. $5^x = \sqrt{125}$

$5^x = \sqrt{5^3}$

$5^x = 5^{\frac{3}{2}}$

$\boxed{x = \frac{3}{2}}$

3. $8^{2+x} = 2$

$2^{3(2+x)} = 2^1$

$3(2+x) = 1$

$6 + 3x = 1$

$-6 \quad -6$

$\frac{3x}{3} = \frac{-5}{3}$

$\boxed{x = -\frac{5}{3}}$

4. $4^{1-x} = 8$

$2^{2(1-x)} = 2^3$

$2 - 2x = 3$

$-2 \quad -2$

$\frac{-2x}{-2} = \frac{1}{-2}$

$\boxed{x = -\frac{1}{2}}$

5. $27^{2x-1} = 3$

$3^{3(2x-1)} = 3^1$

$6x - 3 = 1$

$+3 \quad +3$

$\frac{6x}{6} = \frac{4}{6}$

$\boxed{x = \frac{2}{3}}$

6. $49^{x-2} = 7\sqrt{7}$

$7^{2(x-2)} = 7^1 \cdot 7^{1/2}$

$7^{2x-4} = 7^{3/2}$

$2(2x-4) = \left(\frac{3}{2}\right)2$

$4x - 8 = 3$

$+8 \quad +8$

$\frac{4x}{4} = \frac{11}{4}$

$\boxed{x = \frac{11}{4}}$ or $2\frac{3}{4}$

2.75

7. $25^{2x} = 5^{x+6}$

$5^{2(2x)} = 5^{x+6}$

$4x = x + 6$

$-x \quad -x$

$\frac{3x}{3} = \frac{6}{3}$

$\boxed{x = 2}$

8. $6^{x+1} = 36^{x-1}$

$6^{x+1} = 6^{2(x-1)}$

$\cancel{x} + 1 = 2x - 2$

$-\cancel{x} + 2 \quad -x + 2$

$\boxed{3 = x}$

Composition and Inverse of Functions

In this section we will do a quick review of function notation, and then we will extend it to composite functions.

Review: Evaluate the following if $f(x) = 2x + 1$, $g(x) = \sqrt{x} - 1$, *and* $h(x) = \frac{x+1}{2}$

a) f(3) = $2(3)+1 = 7$ $(3,7)$
$6+1$

b) g(9) = $\sqrt{9} - 1$ $(9,2)$
$3 - 1 = 2$

c) h(5) = $\frac{5+1}{2} = \frac{6}{2} = 3$ $(5,3)$

We can use these same functions to illustrate composite functions. Composite functions are when functions are nested. * start inside, then out

Evaluate:

a) f(g(4)) $= \sqrt{4} - 1$
$= 2 - 1 = 1$
$f(1) = 2(1) + 1$ $\boxed{f(g(4)) = 3}$
$2 + 1 = 3$ $(4,3)$

b) g(f(4)) $f(4) = 2(4) + 1$
$f(4) = 8 + 1 = 9$
$g(9) = \sqrt{9} - 1$
$3 - 1$ $(4,2)$
$\boxed{g(f(4)) = 2}$

Is function notation commutative?
NO, order matters!

c) f(h(7)) $h(7) = \frac{7+1}{2} = \frac{8}{2} = 4$
$f(4) = 2(4) + 1 = 9$
$\boxed{f(h(7)) = 9}$ $(7,9)$

d) f(g(x)) $= 2(\sqrt{x} - 1) + 1$
$2\sqrt{x} - 2 + 1$
$\boxed{f(g(x)) = 2\sqrt{x} - 1}$

e) h(f(x)) $= \frac{(2x+1)+1}{2} = \frac{2x+2}{2} = x+1$
$\boxed{h(f(x)) = x+1}$

Now consider this pair of functions: *Let* $f(x) = x^2$ *and* $g(x) = \sqrt{x}$

Evaluate:

a) f(g(9)) $g(9) = \sqrt{9} = 3$
$f(3) = 3^2 = 9$
$\boxed{f(g(9)) = 9}$ $(9,9)$

b) g(f(7)) $f(7) = 7^2 = 49$
$g(49) = \sqrt{49} = 7$
$\boxed{g(f(7)) = 7}$ $(7,7)$

c) f(g(x)) $= (\sqrt{x})^2$

$\boxed{f(g(x)) = x}$

d) g(f(x)) $= \sqrt{(x^2)}$

$\boxed{g(f(x)) = x}$

f(x) and g(x) are **inverses**. They undo each other.

Two relations, f(x) and g(x), are **Inverses**, if f(g(x)) = g(f(x)) = x.

Inverses are found by switching the __x__ and __y__. An inverse will be a function only if the original relation is one-to-one (it passes the horizontal line test). The notation for the inverse of

$$f(x) \text{ is } f^{-1}(x).$$

Break for Practice: For each problem do the following:

a) Graph f(x)
b) Write the inverse, $f^{-1}(x)$
c) Graph the inverse
d) Is the inverse a function?

1. $f(x) = 2x + 3$

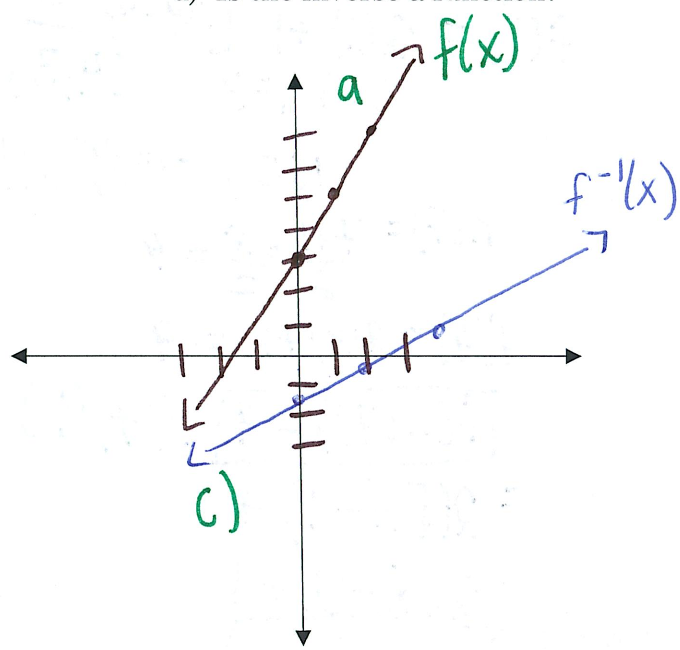
$y = 2x + 3$

inverse: $x = 2y + 3$

(solve for y) $-3 \qquad -3$

$\frac{x}{2} - \frac{3}{2} = \frac{2y}{2}$

$\frac{1}{2}x - \frac{3}{2} = y$ b)



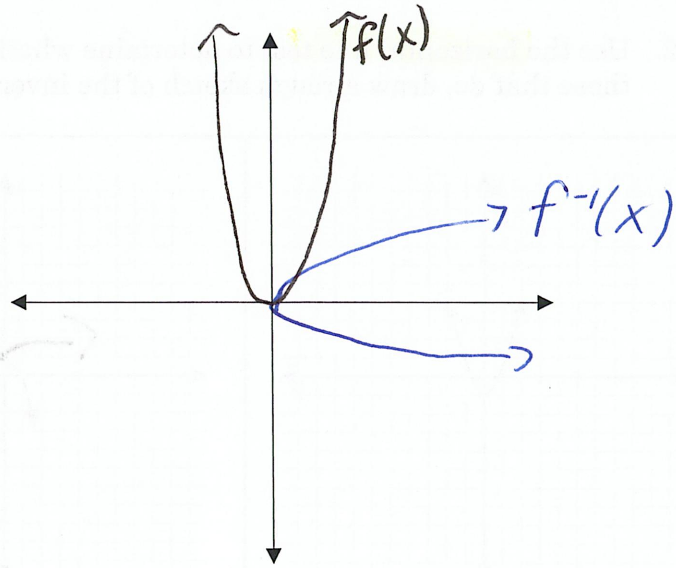
d) yes a function

2. $f(x) = x^4$

$y = x^4$

$\sqrt[4]{x} = \sqrt[4]{y^4}$

$\pm\sqrt[4]{x} = y$



Not a function

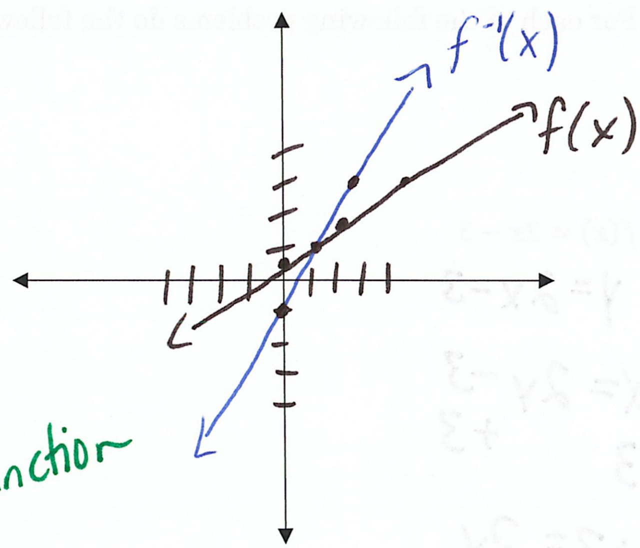
3. $f(x) = \frac{x+1}{2}$

$y = \frac{x+1}{2}$

$2(x) = \left(\frac{y+1}{2}\right)2$

$2x = y + 1$
$\quad -1 \quad -1$

$2x - 1 = y$



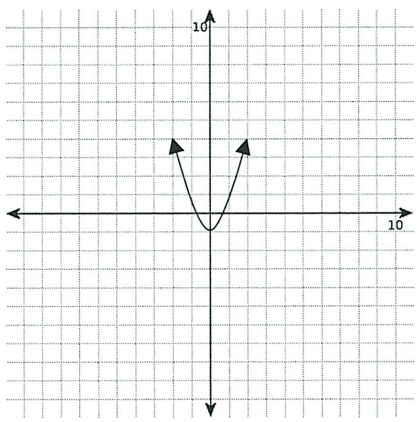
yes, a function

Extended Practice:

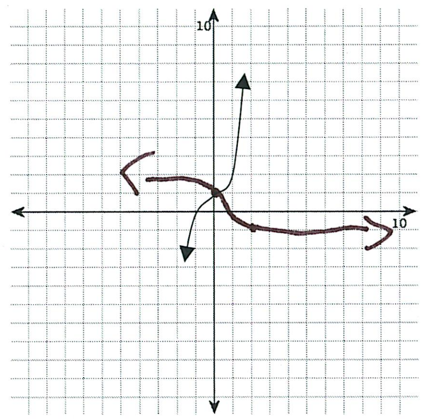
1. Evaluate the following if $f(x) = \frac{x}{2}$, $g(x) = x - 3$, *and* $h(x) = \sqrt{x}$

f(g(8)) $= 8-3$ $= 5$ $f(5) = \boxed{\frac{5}{2}}$	f(g(-5)) $= -5-3$ $= -8$ $f(-8) = \frac{-8}{2} = \boxed{-4}$	f(g(0)) $= 0-3$ $= -3$ $f(-3) = \boxed{\frac{-3}{2}}$	f(g(x)) $= \boxed{\frac{x-3}{2}}$
f(h(9)) $= \sqrt{9}$ $= 3$ $f(3) = \boxed{\frac{3}{2}}$	f(h(4)) $= \sqrt{4}$ $= 2$ $f(2) = \frac{2}{2} = \boxed{1}$	f(h(-4)) $= \sqrt{-4}$ $= 2i$ $f(2i) = \frac{2i}{2} = \boxed{i}$	f(h(x)) $= \boxed{\frac{\sqrt{x}}{2}}$

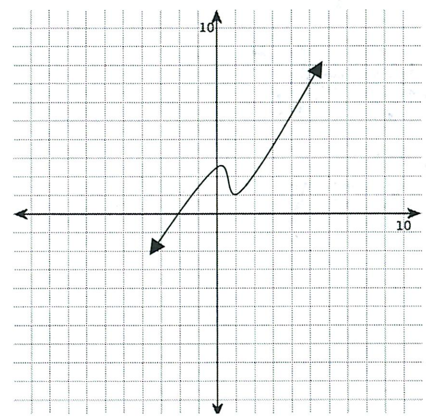
2. Use the horizontal line test to determine whether each function f has an inverse function. For those that do, draw a rough sketch of the inverse on the same set of axes.



Is the inverse a function?
NO



Is the inverse a function?
yes



Is the inverse a function?
NO

For each of the following problems do the following:

a) Graph f(x)
b) Write the inverse, $f^{-1}(x)$
c) Graph the inverse
d) Is the inverse a function?

3. $f(x) = 2x - 3$

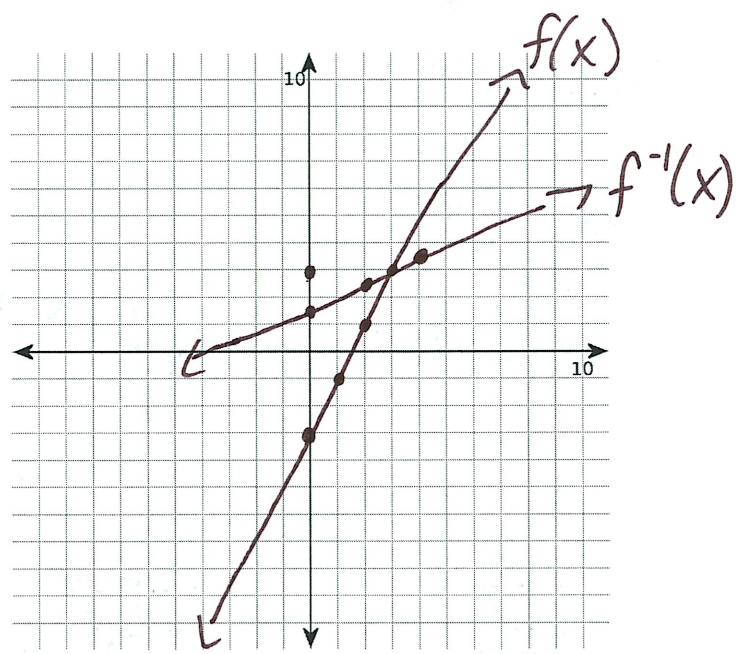
$y = 2x - 3$

$x = 2y - 3$

$+3 \qquad +3$

$\frac{x}{2} + \frac{3}{2} = \frac{2y}{2}$

$\frac{1}{2}x + \frac{3}{2} = y$



yes a fxn

4. $f(x) = \frac{x+6}{3}$

$f(x) = \frac{1}{3}x + 2$

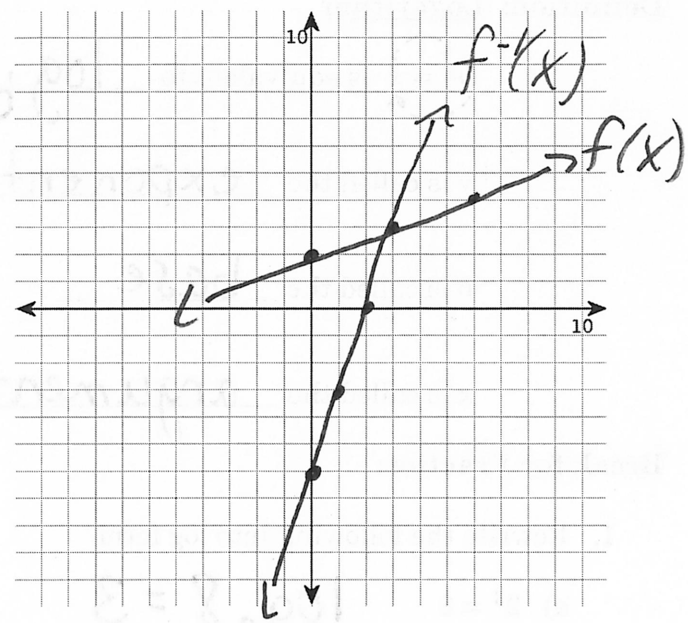
$x = \frac{1}{3}y + 2$

$-2 \qquad -2$

$3(x-2) = (\frac{1}{3}y)3$

$3x - 6 = y$

yes a fxn

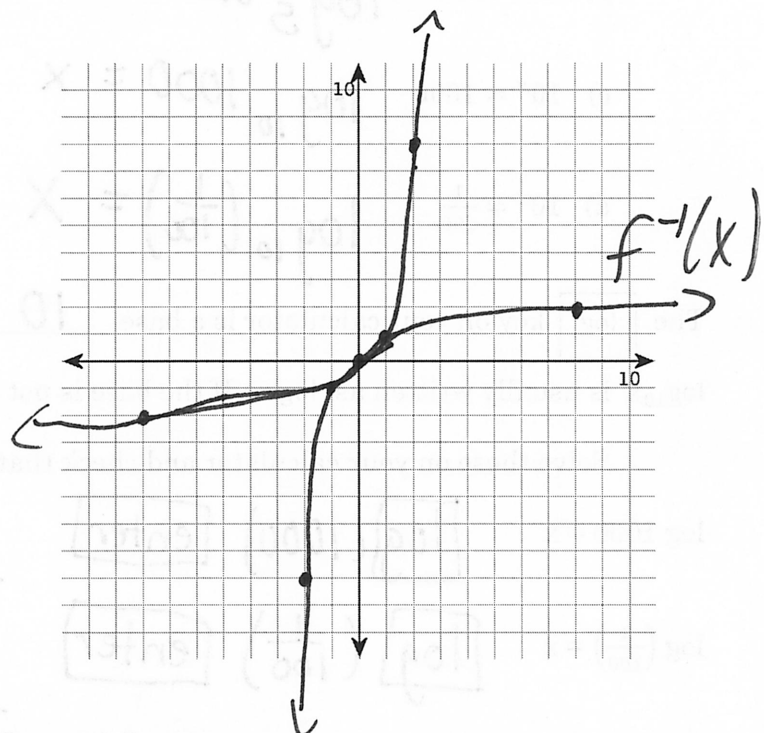


5. $g(x) = x^3$

$\sqrt[3]{x} = \sqrt[3]{y^3}$

$\sqrt[3]{x} = y$

yes a fxn



Definition of Logarithms

The following problems will illustrate the need for a new function.

Solve each problem.

trial and error

a) $3^2 = x$

$9 = x$

b) $\sqrt[3]{x^3} = \sqrt[3]{64}$

$x = 4$

c) $2^x = 13$

$x \approx 3.7$

Definition: Logarithm

$b^y = x$ is equivalent to $\log_b x = y$, x *and* $b > 0$, *and* $b \neq 1$

y is called the exponent or logarithm

b is called the base

x is called the argument

Break for Practice:

1. Rewrite the following into log form.

   a) $2^3 = 8$ $\quad \log_2 8 = 3$

   b) $5^2 = 25$ $\quad \log_5 25 = 2$

   c) $10^x = 1000$ $\quad \log_{10} 1000 = x$

   d) $10^x = \frac{1}{100}$ $\quad \log_{10}\left(\frac{1}{100}\right) = x$

The $\boxed{\text{log}}$ key on your calculator is a base 10 log. This is called the common log.

$\log_{10}x$ is usually written as $\log x$. If the base is not written, then it's a common or base 10 log.

Solve these on your calculator and check that they make sense.

log 1000 = x $\quad \boxed{\log}(1000)\ \boxed{\text{enter}}$ $\quad x = 3$ $\quad 10^3 = 1000$ ✓

$\log\left(\frac{1}{100}\right) = x$ $\quad \boxed{\log}\left(\frac{1}{100}\right)\ \boxed{\text{enter}}$ $\quad x = -2$ $\quad 10^{-2} = \frac{1}{100}$ ✓

Break for Practice:

2. Rewrite the following into exponential form.

   a) $log_4 16 = 2$ $\quad 4^2 = 16$

   b) $log_3\left(\frac{1}{9}\right) = -2$ $\quad 3^{-2} = \frac{1}{9}$