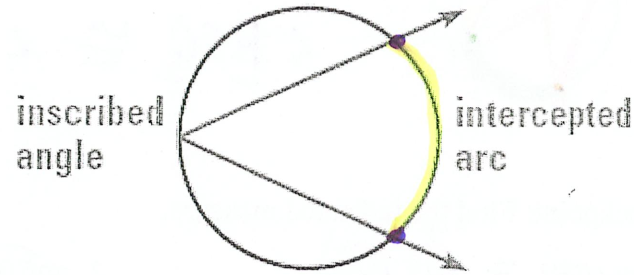
Chapter 10.4: Use Inscribed Angles and Polygons

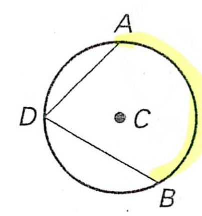
Goal: To be able to use inscribed angles of circles.

An **inscribed angle** is an angle whose vertex is on a circle and whose sides contain chords of the circle. The arc that lies in the interior of an inscribed angle and has endpoints on the angles is the ___intercepted___ of the angle.



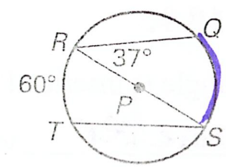
Measure of an Inscribed Angle Theorem (Theorem 10.7)

The measure of an inscribed angle is one half the measure of its intercepted arc.



$m\angle ADB =$ ___$\frac{1}{2} m\widehat{AB}$___

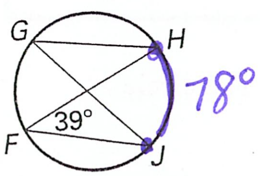
Example #1: Find the indicated measure in ⊙ P.



a. $m\angle S = \frac{1}{2}(60°)$ $\qquad m\widehat{BQ} = 2(37°)$

$m\angle S = 30°$ $\qquad m\widehat{BQ} = 74°$

Example #2: Find $m\widehat{HJ}$ and $m\angle HGJ$. What do you notice about $\angle HGJ$ and $\angle HFJ$.

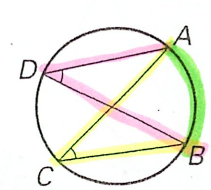


$m\widehat{HJ} = 2(39)° = 78°$

$m\angle HGJ = \frac{1}{2}(78) = 39°$

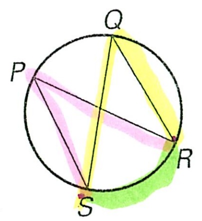
Theorem 10.8:

If two inscribed angles of a circle intercept the same arc, then the angles are congruent.



$\angle ADB \cong$ ___$\angle ACB$___

Example #3: Name two pairs of congruent angles in the figure.

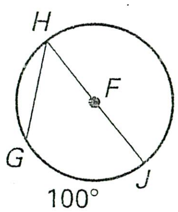


$\angle SPR \cong \angle RQS$

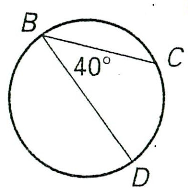
$\angle RSQ \cong \angle PRQ$

Checkpoint: Find the indicated measure.

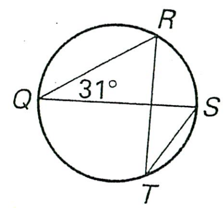
1. $m\angle GHJ$ $= \frac{1}{2}(100)$ 50°



2. $m\overset{\frown}{CD}$ $= 2(40)$ $= 80^\circ$

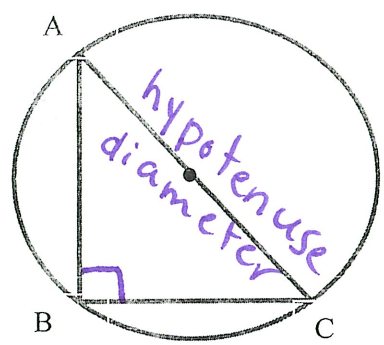


3. $m\angle RTS$ $= 31^\circ$



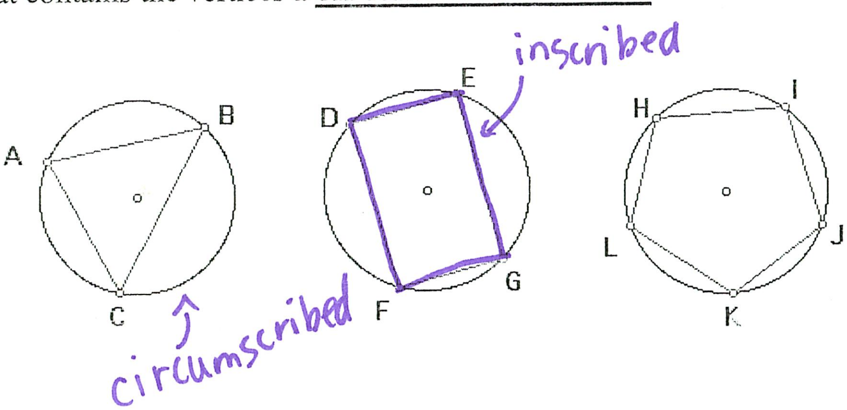
Theorem 10.9:

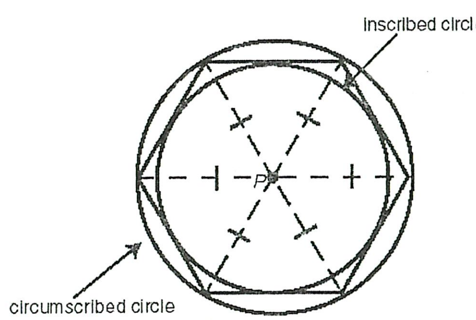
If a right triangle is inscribed in a circle, then the hypotenuse is a diameter of the circle. Conversely, if one side of an inscribed triangle is a diameter of the circle, then the triangle is a right triangle and the angle opposite the diameter is the right angle.



$m\angle ABC =$ 90° iff $\overline{AC}$ is a diameter of $\odot D$.

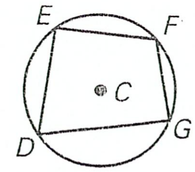
A polygon is an **inscribed polygon** if all of its vertices lie on a circle. The circle that contains the vertices a **circumscribed circle.**





Theorem 10.10:

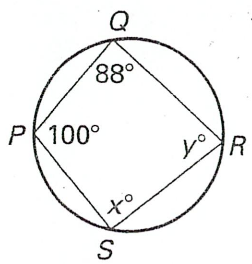
A quadrilateral can be inscribed in a circle iff its opposite angles are supplementary (180°).



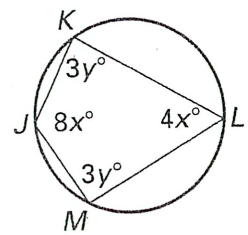
D, E, F and G lie on $\odot C$ iff $m\angle D + m\angle F =$ 180°

Example #4: Find the value of each variable.

a.



b.



$\angle Q + \angle S = 180°$

$88 + x = 180$

$x = 92°$

$\angle P + \angle R = 180$

$100 + y = 180$

$y = 80°$

$3y + 3y = 180$

$6y = 180$

$y = 30°$

$8x + 4x = 180$

$12x = 180$

$x = 15°$

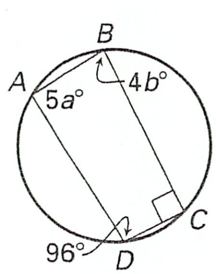
Example #5: A right triangle is inscribed in a circle. The radius of the circle is 5.6 cm. What is the length of the hypotenuse of the right triangle?

(diameter)

hypotenuse = 2(5.6)

$\boxed{\text{hypotenuse} = 11.2 \text{ cm}}$

Checkpoint: Find the values of *a* and *b*.



$4b + 96 = 180$

$-96 \quad -96$

$\frac{4b}{4} = \frac{84}{4}$

$b = 21$

$5a + 90 = 180$

$-90 \quad -90$

$\frac{5a}{5} = \frac{90}{5}$

$a = 18$