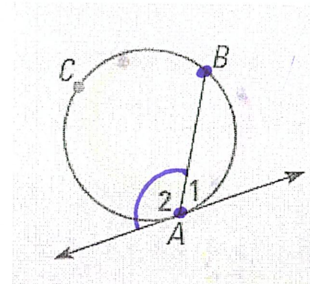
Chapter 10.5: Apply Other Angle Relationships in Circles

Goal: Be able to find the measures of angles inside or outside a circle.

Theorem 10.11:

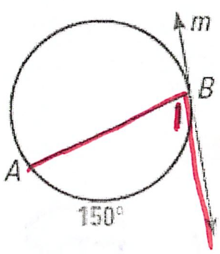
If a tangent and a chord intersect at a point on a circle, then the measure of each angle formed is half the measure of its intercepted arc.



$m\angle 1 = \frac{1}{2} m\overset{\frown}{AB}$ $m\angle 2 = \frac{1}{2} m\overset{\frown}{BCA}$

Example #1: Line *m* is tangent to the circle. Find the measure of the red angle or arc.

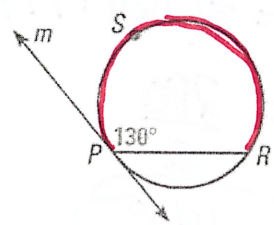
a.



$m\angle 1 = \frac{1}{2}(150)$

$75°$

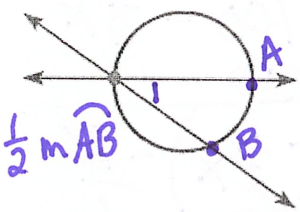
b.



$m\overset{\frown}{PSR} = 2(130)$

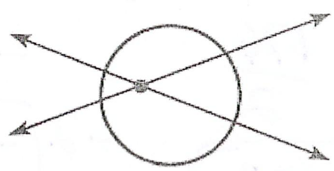
$= 260°$

Intersecting Lines and Circles: If two lines intersect a circle, there are three places where the lines can intersect.

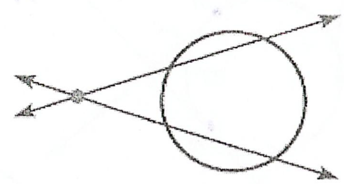


$m\angle 1 = \frac{1}{2} m\overset{\frown}{AB}$

on the circle



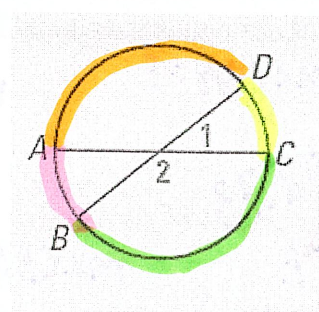
inside the circle



outside the circle

Angles Inside the Circle Theorem (Theorem 10.12):

If two chords intersect inside a circle, then the measure of each angle is one half the sum of the measures of the arcs intercepted by the angle and its vertical angle.

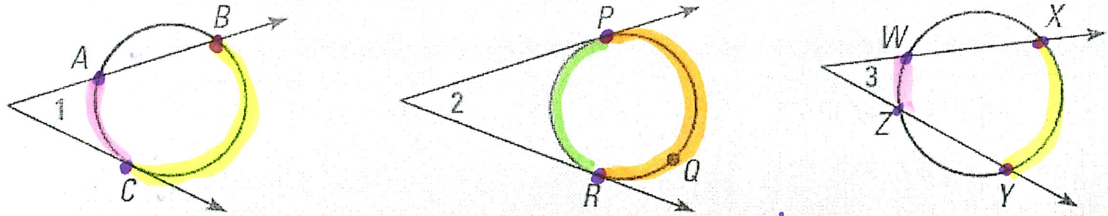


$m\angle 1 = \frac{1}{2}(m\overset{\frown}{DC} + m\overset{\frown}{AB})$

$m\angle 2 = \frac{1}{2}(m\overset{\frown}{AD} + m\overset{\frown}{BC})$

Angles Outside the Circle Theorem (Theorem 10.13):

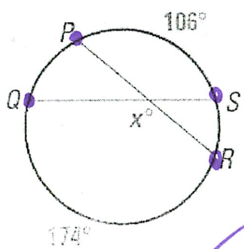
If a tangent and a secant, two tangents, or two secants intersect outside a circle, then the measure of the angle formed is one half the difference of the measures of the intercepted arcs.



$m\angle 1 =$ $\frac{1}{2}(m\overset{\frown}{BC} - m\overset{\frown}{AC})$ $m\angle 2 =$ $\frac{1}{2}(m\overset{\frown}{PQR} - m\overset{\frown}{PR})$ $m\angle 3 =$ $\frac{1}{2}(m\overset{\frown}{XY} - m\overset{\frown}{WZ})$

Example #2: Find the value of x.

a. +

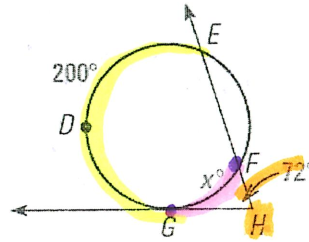


$x = \frac{1}{2}(106 + 174)$

$x = \frac{280}{2}$

$x = 140°$

b.



$m\angle H = \frac{1}{2}(m\overset{\frown}{EDG} - m\overset{\frown}{FG})$

$72° = \frac{1}{2}(200 - x)$

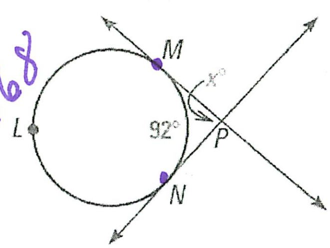
$72° = 100 - \frac{1}{2}x$

$-100 \quad -100$

$-2(-28) = (-\frac{1}{2}x) \cdot -\frac{2}{1}$

$\boxed{56° = x}$

c. −



268

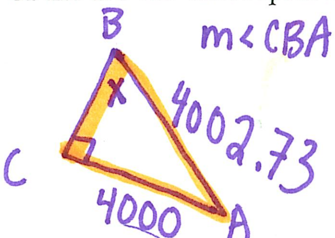
$360 - 92 = 268$

$x = \frac{1}{2}(268 - 92)$

$= \frac{176}{2}$

$x = 88°$

Example #3: You are on top of Mount Rainier on a clear day. You are about 2.73 miles above sea level. Find the measure of the arc $\overset{\frown}{CD}$ that represents the part of Earth that you can see.



$m\angle CBA = ?$

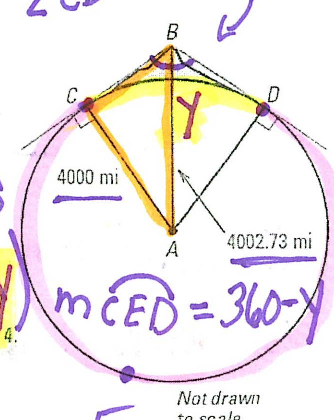
$\sin^{-1}\left(\frac{4000}{4002.73}\right) \approx 87.9°$

$m\angle CBA$

$m\angle CBD = 2(87.9)$

$m\angle CBD = 175.8°$

$\angle CBD$ 175.8



$\overset{\frown}{CED}$ $\overset{\frown}{CD}$

$175.8 = \frac{1}{2}(360 - y - y)$

$m\overset{\frown}{CED} = 360 - y$

4.

$175.8 = \frac{1}{2}(360 - 2y)$

$175.8 = 180 - y$

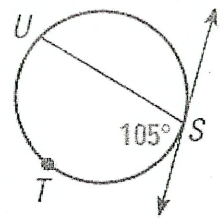
$-180 \quad -180$

$-4.2 = -y \div -1$

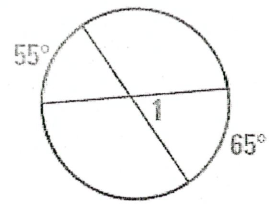
$y = \boxed{4.2° = m\overset{\frown}{CD}}$

Checkpoint:

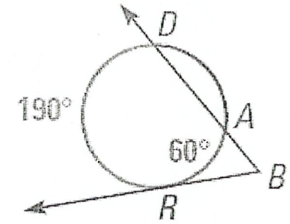
1. $m\overset{\frown}{STU}$



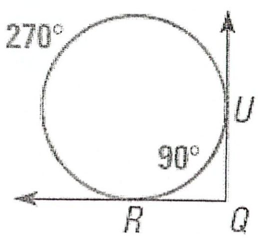
2. $m\angle 1$



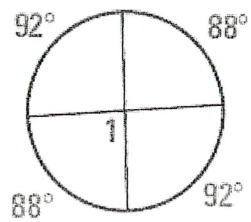
3. $m\angle DBR$



4. $m\angle RQU$



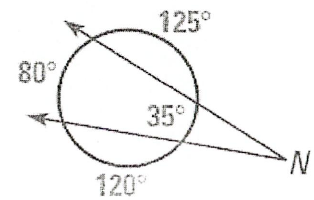
5. $m\angle 1$



$m\angle 1 = \frac{1}{2}(88+88)$

$m\angle 1 = 88°$

6. $m\angle N$



$m\angle N = \frac{1}{2}(80-35)$

$= \frac{1}{2}(45)$

$m\angle 4 = 22.5°$