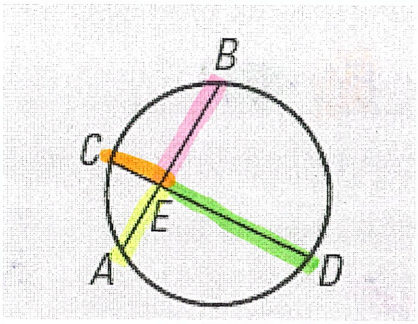
Chapter 10.6: Find Segment Lengths in Circles

** When two chords intersect in the interior of a circle, each chord is divided into two segments that are **

called segments of the chord

Segments of Chords Theorem (Theorem 10.14):

If two chords intersect in the interior of a circle, then the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord.



EA · EB = CE · ED

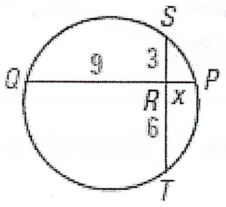
Example #1: Chords $\overline{ST}$ and $\overline{PQ}$ intersect inside the circle. Find the value of x.

ST PQ

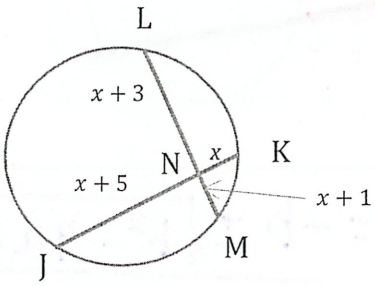
$3 \cdot 6 = 9x$

$18 = 9x$

$2 = x$



Example #2: Find ML and JK



LM JK

$(x+3)(x+1) = (x+5)x$

$x^2 + x + 3x + 3 = x^2 + 5x$

$x^2 + 4x + 3 = x^2 + 5x$

$-x^2 \quad -4x \qquad -x^2 \quad -4x$

$3 = x$

x+3 x+1

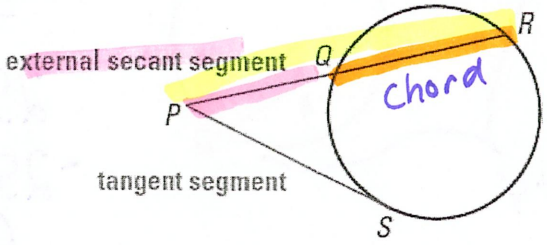
$ML = 6 + 4$

$ML = 10$

$JK = \underset{8}{x+5} + \underset{3}{x}$

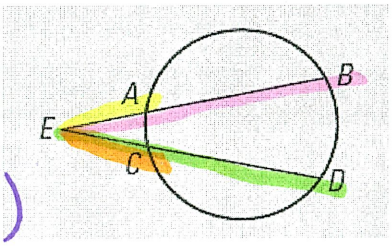
$JK = 11$

A secant segment is a segment that contains a chord of a circle, and has exactly one endpoint outside the circle. The part of the secant segment that is outside the circle is called an external secant segment.



Segments of Secants Theorem (Theorem 10.15):

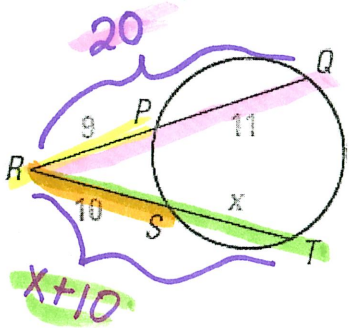
If two secant segments share the same endpoint outside a circle, then the product of the lengths of one secant segment and its external segment equals the product of the lengths of the other secant segment and its external segment.



part(total) = part(total)

EA · EB = EC · ED

Example #3: Find the value of x.



part total part total
RP(RQ) = RS(RT)

9(20) = 10(x+10)

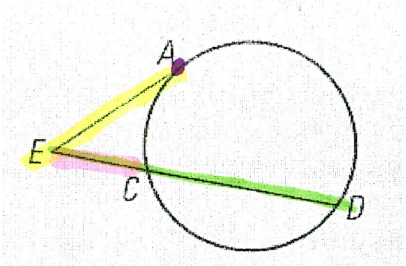
180 = 10x + 100
−100 −100

$\frac{380}{10} = \frac{10x}{10}$

x = 8

Segments of Secants and Tangents Theorem (Theorem 10.16):

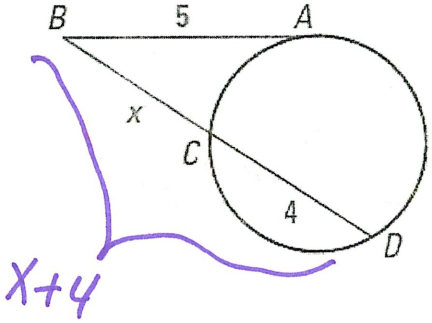
If a secant segment and a tangent segment share an endpoint outside a circle, then the product of the lengths of the secant segment and its external segment equals the square of the lengths of the tangent segment.



part total

$(EA)^2$ = EC · ED

Example #4: Find the value of x.



part part(total)

$BA^2 = BC(BD)$

$5^2 = x(x+4)$

$25 = x^2 + 4x$

−25 −25

$0 = x^2 + 4x - 25$

a b c

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-4 \pm \sqrt{4^2 - 4(1)(-25)}}{2(1)}$

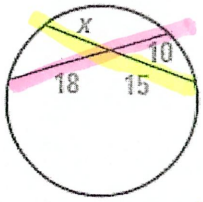
$x = \frac{-4 \pm \sqrt{116}}{2}$

$x \approx 3.39$ $x = -7.39$

Checkpoint: Find the value of x.

1 pg

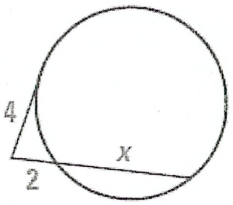
1.



$15x = 18 \cdot 10$

$x = 12$

2.



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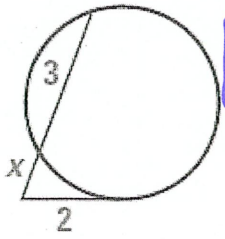


$x = 1, -4$

$2^2 = x(x+3)$

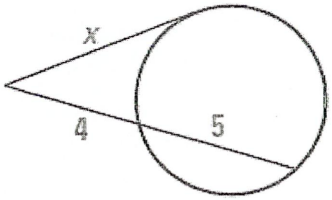
$4 = x^2 + 3x$

$-4 \quad -4$

$0 = x^2 + 3x - 4$

$0 = (x-1)(x+4)$

4.



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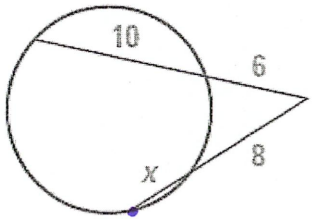


$6(16) = 8(x+8)$

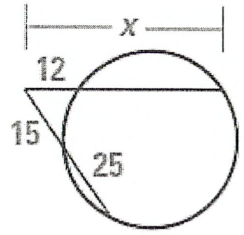
$96 = 8x + 64$

$-64 \quad -64$

$32 = 8x$

$x = 4$

6.

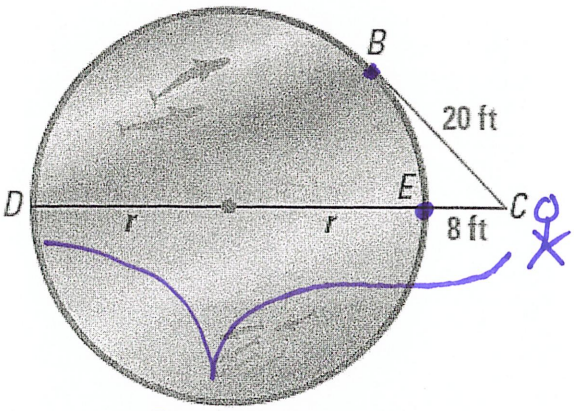


Example #5: You are standing at point C, about 8 feet from a circular aquarium tank. The distance from you to a point of tangency on the tank is about 20 feet. Estimate the radius of the tank.

part total

$CB^2 = CE(CD)$

$20^2 = 8(2r+8)$



$2r + 8$