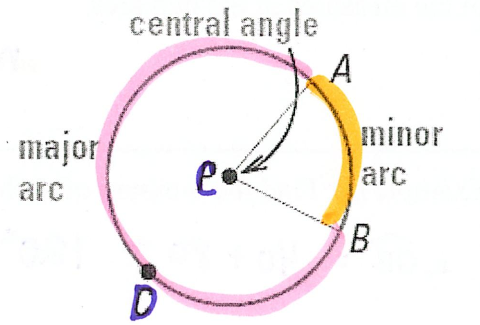
Chapter 10.2: Find Arc Measures

Goal: Be able to use angle measures to find arc measures

Arc Measures:

A **central angle** of a circle is an angle whose vertex is the center of the circle.



If $m\angle ACB$ is less than 180°, then the points on $\odot C$ that lie in the interior of $\angle ACB$ form a minor arc with endpoints A and B. (The measure of a minor arc is the measure of its central angle.) $m\overset{\frown}{AB} = m\angle APB$

Naming: Minor arcs are named by their endpoints

$\angle ACB$ is named $\overset{\frown}{AB}$

The points on $\odot C$ that do not lie on minor arc $\overset{\frown}{AB}$ form a major arc with endpoints A and B. (The measure of the entire circle is 360°. The measure of a major arc is the difference between 360° and the measure of the related minor arc)

Naming: Major arcs and semicircles are named by endpoints and a point on the arc (3 pts)

major arc of $\angle ACB$ is $\overset{\frown}{ADB}$

A semicircle is an arc with endpoints that are the endpoints of a diameter.

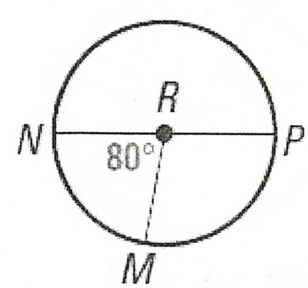
$m = 180°$

Example #1: For each arc of $\odot R$, Identify as a minor arc, major arc or semicircle and find its measure.

a. $\overset{\frown}{MN}$ minor, 80°

b. $\overset{\frown}{MPN}$ major arc, 360 − 80 = 280°

c. $\overset{\frown}{PMN}$ semicircle, 180°

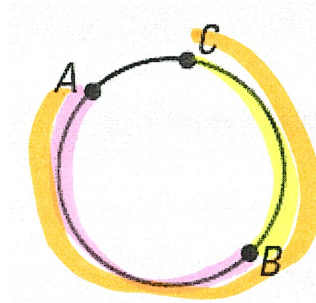


Two arcs of the same circle are **adjacent** if they intersect at exactly one point. You can add the measures of adjacent arcs.

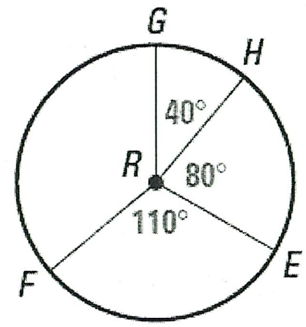
Arc Addition Postulate (Postulate 26):

The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs.

$m\widehat{ABC} = m\widehat{AB} + m\widehat{BC}$



Example #2: Find the measure of each arc.



a. $\widehat{GE}$ = $40 + 80 = 120°$

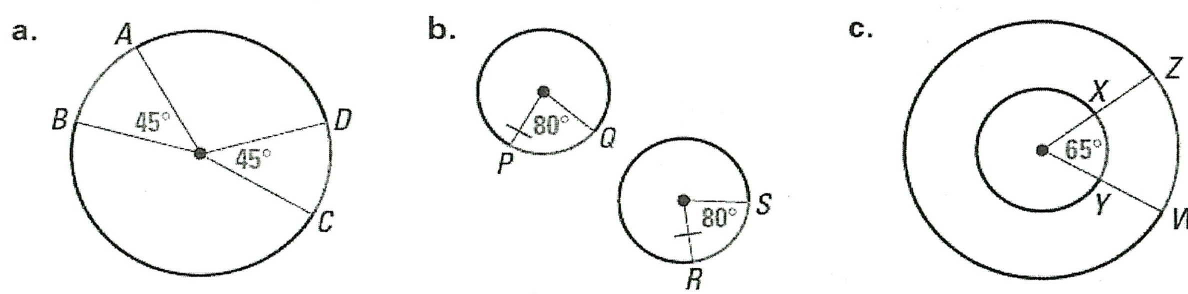
b. $\widehat{GEF}$ = $40 + 80 + 110° = 230°$

c. $\widehat{GF}$ = $360 - 230 = 130°$

d. $\widehat{EFG}$ = $130 + 110 = 240°$

Two circles are **congruent circles** if they have the same radius. Two arcs of the same circle or of congruent circles are **congruent arcs** if they have the same measure. So, two minor arcs of the same circle or of congruent circles are congruent if their central angles are congruent.

Example #3: Are the given arcs congruent?



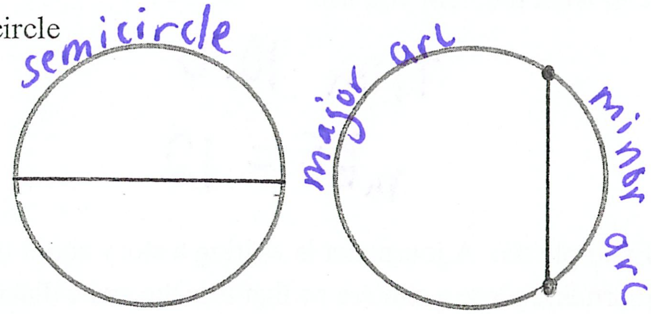
a. yes $\widehat{AB} \cong \widehat{CD}$

b. yes

c. NO, not congruent circles

Chapter 10.3: Apply Properties of Chords

Goal: I will be able to use relationships of arcs and chords in a circle



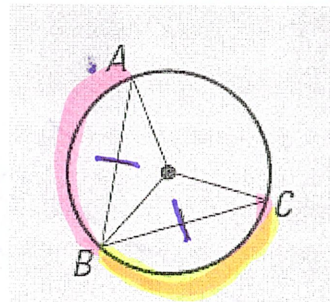
Recall: A chord is a segment with endpoints on a circle. Because its endpoints lie on the circle, any chord divides the circle into two arcs. A **diameter** divides the circle into two ___Semicircles___.

Any other chord divides a circle into a **minor arc** and a **major arc.**

Theorem 10.3:

In the same circle, or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent.

$\overline{AB} \cong \overline{BC}$ iff $\overset{\frown}{AB} \cong \overset{\frown}{BC}$

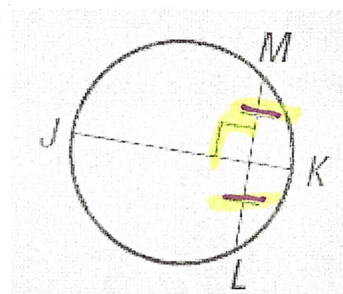


Theorem 10.4:

If one chord is a perpendicular bisector of another chord, then the first chord is a diameter.

If ___$\overline{JK}$___ is a perpendicular bisector of ___$\overline{ML}$___,

then ___$\overline{JK}$___ is a diameter of the circle.

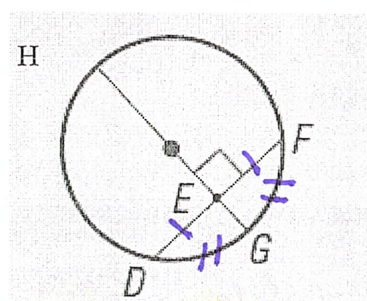


Theorem 10.5:

If a diameter of a circle is perpendicular to a chord, then the diameter bisects the chord and its arc.

If ___$\overline{HG}$___ is a diameter and ___$\overline{HG} \perp \overline{DF}$___,

then ___$\overline{ED} \cong \overline{EF}$___ and ___$\overset{\frown}{ED} \cong \overset{\frown}{EF}$___,

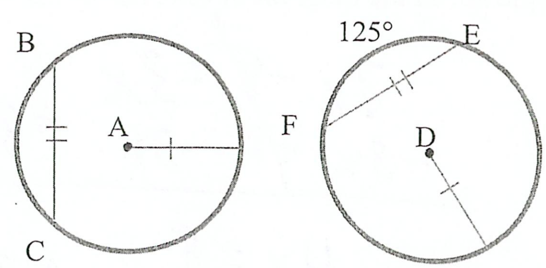


Example #1: In the diagram, $\odot A \cong \odot D$, $\overline{BC} \cong \overline{EF}$, and $m\overset{\frown}{EF} = 125°$. Find $m\overset{\frown}{BC}$.

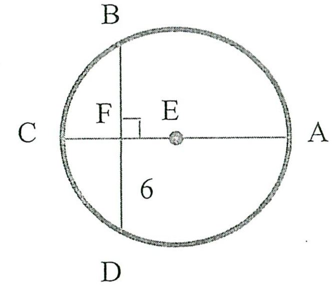
$m\overset{\frown}{BC} = 125°$

$2 \cong \odot$'s, $2 \cong$ chords

Thm 10.3



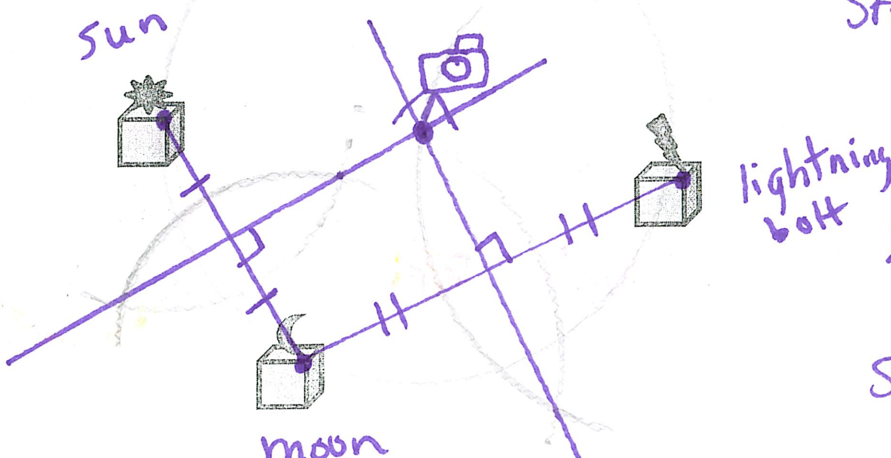
Example #2: Use the diagram of $\odot E$ to find the length of $\overline{BD}$ ..
Tell what theorem you use



Thm 10.5

$m\overline{BD} = 12$

Example #3: A journalist is writing a story about three sculptures, arranged as shown. Where should the journalist place a camera so that it is the same distance from each sculpture?

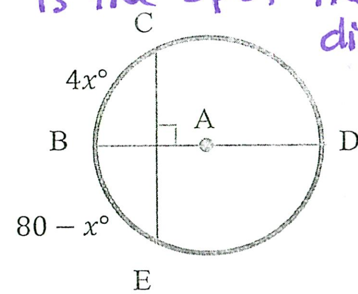


Step 1: draw lines connecting the 3 sculptures

Step 2: bisect each lines

Step 3: where 2 ⊥ bisectors cross is the spot the same distance away

Example #4: Find the measures of $\widehat{CB}$, $\widehat{BE}$, and $\widehat{CE}$



$m\widehat{CB} = 4(16) = 64°$

$m\widehat{BE} = 64°$

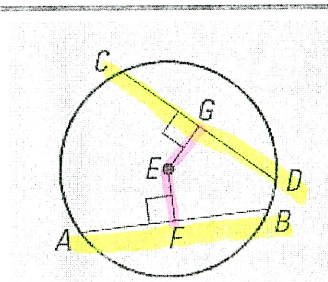
$m\widehat{CE} = 64 + 64 = 128°$

$$\begin{aligned} 4x &= 80 - x \\ +x &\quad\ \ +x \\ \hline 5x &= 80 \\ x &= 16 \end{aligned}$$

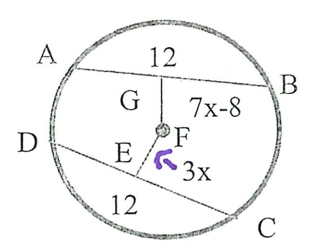
Theorem 10.6:

In the same circle, or in congruent circles, two chords are congruent if and only if they are equidistant from center.

$\overline{AB} \cong \overline{CD}$ iff $EG = EF$



Example #5: In the diagram of $\odot F$, $AB = CD = 12$. Find EF.



$$\begin{aligned} 7x - 8 &= 3x \\ -3x + 8 &\quad -3x \quad +8 \\ \hline 4x &= 8 \qquad x = 2 \end{aligned}$$

$EF = 3(x)$
$= 3(2)$
$\boxed{EF = 6}$