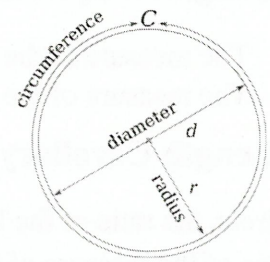
Chapter 11.4: Circumference and Arc Length

Circumference of a Circle (Theorem 11.8):

The circumference C of a circle is $C = 2\pi r$ or πd

Where d is the diameter of the circle and r is the radius of the circle



Exact Measure: putting your answer in terms of π (not rounding)

Example) the circumference of a circle with diameter 6

$C = 6\pi$ ← exact $C \approx 18.85$ rounded or approx.

Example #1: Find the indicated measure.

a) Circumference of a circle with radius 9 cm

$C = 2\pi r$

$r = 9$

$C = 2\pi(9)$

$\boxed{C \approx 56.55 \text{ cm}}$

b) Radius of a circle with circumference 26 m

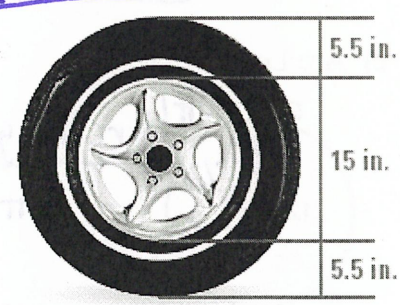
r

$r = ?$ $C = 26$ m

$\frac{26}{(2\pi)} = \frac{2\pi r}{2\pi}$

$\boxed{4.14 \text{ m} \approx r}$

Example #2: The dimensions of a car tire is shown at the right
To the nearest foot, how far does the tire travel when it makes 15 revolutions?



1 rev = 1 full time around (circumference

$d = 5.5 + 15 + 5.5$

$d = 26$ in

$C = 26\pi$

$C \approx 81.68$ in

distance around circle

how many complete circles

Distance $= 81.68 \times 15$

distance $= 1225.2$ in

convert to feet: $1225.2 \text{ in} \times \frac{1 \text{ ft}}{12 \text{ in}}$

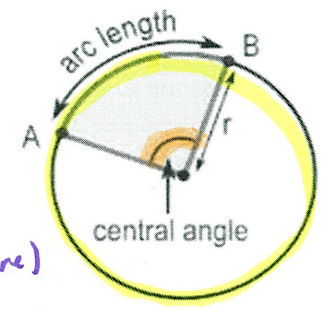
≈ 102.1

$\boxed{102 \text{ ft}}$

Central Angle: A central angle of a circle is an angle whose vertex is the center of the circle.

Arc Length: is a portion of the circumference of circle.

- The measure of the arc is measured in degrees
- The measure of the length is measured in linear units



Arc Length Corollary

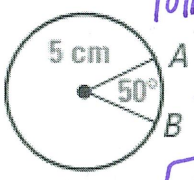
In a circle, the ratio of the length of a given arc to the circumference is equal to the ratio of the measure of the arc to 360°

$\frac{\text{Arc Length of } \widehat{AB}}{2\pi r} = \frac{m\widehat{AB}}{360°} \rightarrow$ Arc Length of $\widehat{AB} = \frac{m\widehat{AB}}{360°} \cdot 2\pi r$

circumference (angle measure)

Example #3: Find the indicated measure.

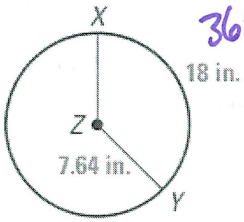
a) Arc Length of $\widehat{AB}$



$10\pi\left(\frac{\text{Arc Length } \widehat{AB}}{2\pi 5}\right) = \left(\frac{50°}{360°}\right)10\pi$

$\boxed{\text{Arc length } \widehat{AB} = 4.36 \text{ cm}}$

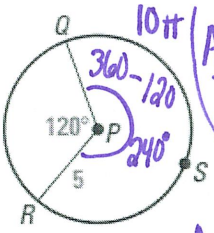
b) $m\widehat{XY}$



$360°\left(\frac{18}{2\pi(7.64)}\right) = \left(\frac{m\widehat{XY}}{360°}\right)360$

$\boxed{m\widehat{XY} = 135°}$

c) Arc Length of $\widehat{QSR}$



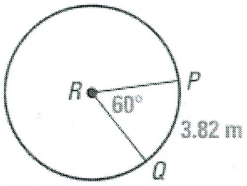
360 − 120

240°

$10\pi\left(\frac{\text{Arc length } \widehat{QSR}}{2\pi 5}\right) = \left(\frac{240°}{360°}\right)10\pi$

Arc length $\widehat{QSR} = \boxed{20.9 \text{ units}}$

d) Circumference of $\odot R$



$\frac{3.82}{C} = \frac{60}{360}$

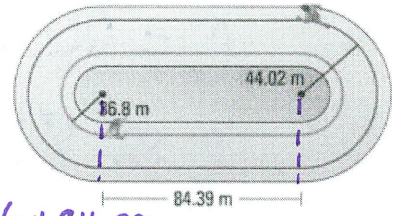
$\frac{60C}{60} = \frac{1375.2}{60}$

$\boxed{C = 22.92 \text{ m}}$

Example #4: The curves at the ends of the track show are 180° arc of circles. The radius of the arc for a runner on the ~~outside~~ inside path is 36.8 meters. About how far does this runner travel to go once around the track? Round to the nearest tenth of a meter.

$(2\pi 36.8)\left(\frac{\text{Arc length (inside)}}{2\pi(36.8)}\right) = \left(\frac{180°}{360°}\right)(2\pi\ 36.8)$

Distance = arc + straight + arc + straight



Arc length (inside) = 115.6

D = 115.6 + 84.39 + 115.6 + 84.39

D = 399.98 m

$\boxed{D = 400 \text{ m}}$