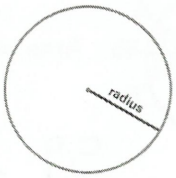
Chapter 11.5: Areas of Circles and Sectors

Area of a Circle (Theorem 11.9):

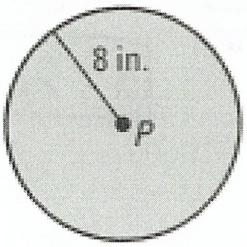
$A =$ πr^2

* if given diameter $\frac{d}{2} = r$



Example #1: Find the indicated measure

a) Find the area of $\odot P$.

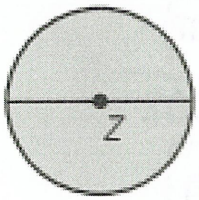


$A = \pi 8^2$

$A = 64\pi \text{ in}^2 \text{ (exact)}$

$A \approx 201.06 \text{ in}^2$

b) Find the diameter of $\odot Z$ if the $A = 96cm^2$



$\frac{96}{\pi} = \frac{\pi r^2}{\pi}$

$\sqrt{30.56} = \sqrt{r^2}$

$5.53 \text{ cm} = r$

$2(5.53) = d$

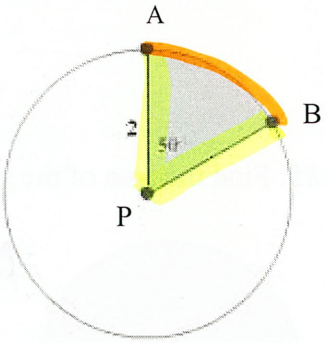
$11.06 \text{ cm} = d$

Sector of a Circle: is the region bounded by two radii of the circle and their intercepted arc.

In the diagram, sector APB is bounded by… $\overline{AP}$, $\overline{BP}$, arc length AB

Area of a Sector (Theorem 10.10):

The ratio of the area of a sector of a circle to the area of the whole circle (πr^2) is equal to the ratio of the measure of the intercepted arc to 360°

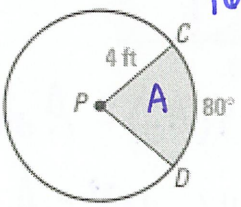


$$\frac{\text{Area of sector APB}}{\pi r^2} = \frac{m\overset{\frown}{AB}}{360°}$$

(total) A ← angle (degrees)

Example #2: Find the area of the shaded area shown.

a)

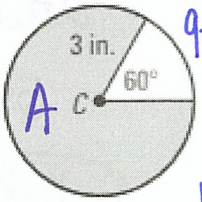


$16\pi \left(\frac{A}{\pi 4^2}\right) = \left(\frac{80}{360}\right) 16\pi$

$A = 11.17 \text{ ft}^2$

b)

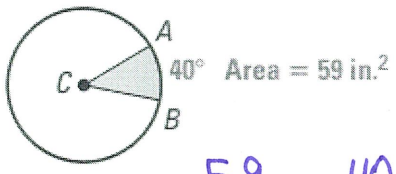
$360 - 60 = 300°$



$9\pi \left(\frac{A}{\pi 3^2}\right) = \left(\frac{300}{360}\right) 9\pi$

$A = 23.56 \text{ in}^2$

Example #3: Find the radius of $\odot C$.



$\frac{59}{\pi r^2} = \frac{40}{360}$

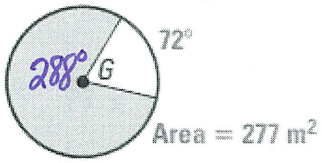
$\frac{40\pi r^2}{(40\pi)} = \frac{21240}{(40\pi)}$

$\sqrt{r^2} = \sqrt{169.02}$

$r \approx 13$ in

Example #4: Find the diameter of $\odot G$.

$360 - 72 = 288$



$\frac{277}{\pi r^2} = \frac{288}{360}$

$\frac{288\pi r^2}{288\pi} = \frac{99720}{(288\pi)}$

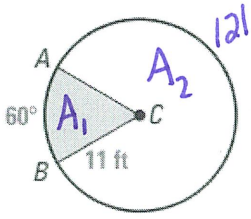
$\sqrt{r^2} = \sqrt{110.21}$

$r \approx 10.50$

$d = (10.5)2$

$d = 21$ m

Example #5: Find the areas of the sectors formed by $\angle ACB$



$121\pi \left(\frac{A_1}{\pi 11^2}\right) = \left(\frac{60}{360}\right) 121\pi$

$A_1 = 63.36\ ft^2$

total

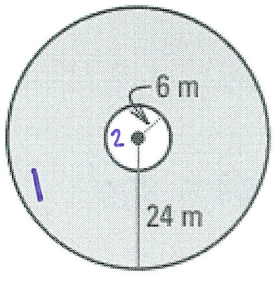
$\pi 11^2 - A_1 = A_2$

$380.13 - 63.36 = A_2$

$316.77\ ft^2 = A_2$

Example #5: Find the area of the shaded region.

a)



$A_{large} - A_{small} = A_{shaded}$
1 2

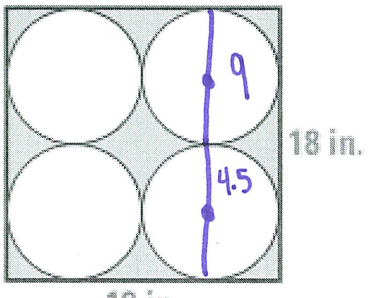
$\pi 24^2 - \pi 6^2 = A_{shaded}$

$576\pi - 36\pi = A_{shaded}$

$540\pi = A_{shaded}$

$1696.46\ m^2 = A_{shaded}$

b)



$A_{\square} - 4(A_{\circ}) = A_{shaded}$

$18^2 - 4(\pi 4.5^2) = A$

$324 - 4(63.62) =$

$324 - 254.48 =$

$69.52 = A_{shaded}$