Chapter 11.6: Areas of Regular Polygons

Center of a polygon: The center of a polygon is the center of its circumscribed circle.

Example: P

Radius of a polygon: The radius of a polygon is the radius of its circumscribed circle.

Example: $\overline{PN}$

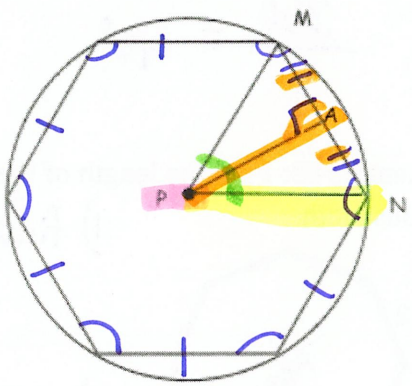
(height)

Apothem of a polygon: The distance from the center to any side of the polygon.

Example: $\overline{PA}$ (connects the center to the midpoint of a side. ⊥)

Central angle of a regular polygon: A central angle of a regular polygon is an angle formed by two radii drawn to consecutive vertices of the polygon.

Example: $\angle MPN$ $\frac{360}{n}$ n = # of sides



Identify $\triangle MPN$ by its sides.

isosceles

How does the apothem relate to $\triangle MPN$?

it's the height

$\angle MPA \cong \angle NPA$ and $\overline{MA} \cong \overline{NA}$

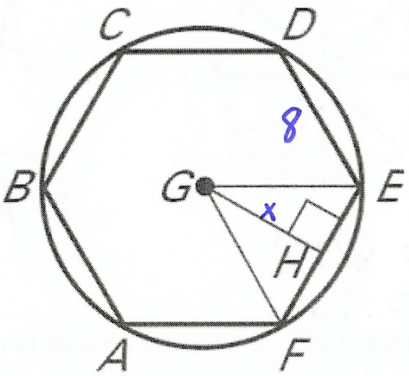
Example #1: In the diagram, ABCDEF is a regular hexagon inscribed in $\odot G$. If DE= 8cm, find each of the following.

a) $m\angle EGF =$ central ∠ n=6 $\frac{360}{6} = 60°$

b) $m\angle EGH = \frac{60}{2} = 30°$

c) $m\angle HEG =$ 90 - 30 = 60°

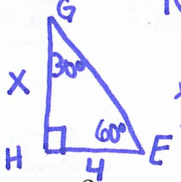
d) FE = 8 cm



e) HE = 4 cm

f) GH = special rt Δ's $4\sqrt{3} \approx 6.93$ cm

or trig $\frac{\tan 60° = \frac{x}{4}}{1}$ $x = 4 \tan 60$ $x = 6.93$ cm



g) What is the perimeter of the hexagon?

8 × 6 = 48 cm

Example #2: Find the measure of a central angle of a regular polygon with the given number of sides.

a) 9 sides

$\frac{360}{9} = 40°$

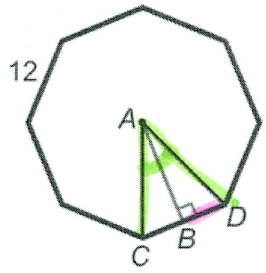
b) 15 sides

$\frac{360}{15} = 24°$

c) 30 sides

$\frac{360}{30} = 12°$

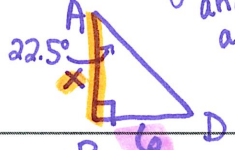
Example #3: Find the length of the apothem in the regular octagon. Round your answer to the nearest thousandth.



1) find central angle

$\frac{360}{8} = 45°$

2) pull out right △ and divide central angle by 2



$\frac{45}{2} = 22.5°$

3) cut side length in half for base $\frac{12}{2} = 6$

4) use trig to solve for apothem

$\frac{\tan 22.5}{1} = \frac{6}{x}$

$x \tan(22.5) = 6$

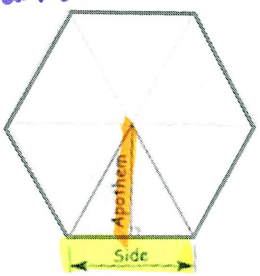
$\frac{x \tan 22.5}{\tan 22.5} = \frac{6}{\tan 22.5}$

$x = 14.49$

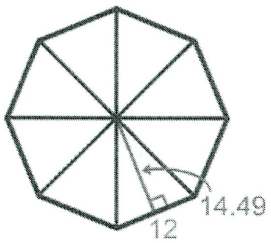
Area of a Regular Polygon (Theorem 11.11):

The area of a regular n-gon with side length s is half the product of the apothem a and the perimeter P $(n \cdot s)$

$A = \frac{1}{2}aP$ or $A = \frac{aP}{2}$ $\rightarrow$ $A = \frac{a(n \cdot s)}{2}$ P



Example # 4: Find the area of the regular octagon. Round your answer to the nearest thousandth.



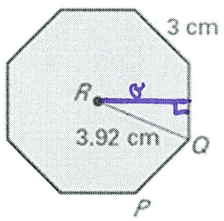
$P = 8(12)$

$P = 96$

$A = \frac{14.49(96)}{2}$

$A = 695.52 \text{ units}^2$

Example #5: A wooden coaster is a regular octagon with 3 cm sides and a radius of about 3.92 cm. What is the area of the coaster? Round your answer to the nearest thousandth.



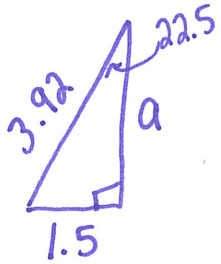
$P = 3 \cdot 8$

$P = 24$

central ∠

$\frac{360}{8} = 45$

$\frac{45}{2} = 22.5$



$\frac{\tan 22.5}{1} = \frac{1.5}{a}$

$\frac{a \tan 22.5}{\tan 22.5} = \frac{1.5}{\tan 22.5}$

$a = 3.62$ cm

or

$1.5^2 + a^2 = 3.92^2$

$-1.5^2 \quad -1.5^2$

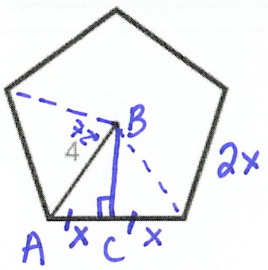
$\sqrt{a^2} = \sqrt{13.12}$

$A = \frac{a \cdot P}{2}$

$A = \frac{3.62(24)}{2}$

$A = 43.44 \text{ cm}^2$

Example #6: Find the area of the regular pentagon with radius 4. Round your answer to the nearest thousandth.

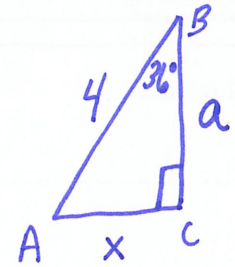


P

Central ∠

$\frac{360}{5} = 72°$

$\frac{72}{2} = 36°$



$\frac{\sin 36}{1} = \frac{x}{4}$

$4 \sin 36 = x$

$2.35 = x$

$\frac{\cos 36}{1} = \frac{a}{4}$

$4 \cdot \cos(36) = a$

$3.24 = a$

$2(2.35) = s$

$4.7 = s$

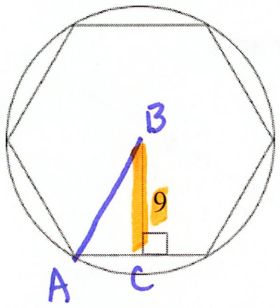
$P = 4.7(5) = 23.5$

$A = \frac{a \cdot P}{2}$

$A = \frac{3.24(23.5)}{2}$

$A = 38.07 u^2$

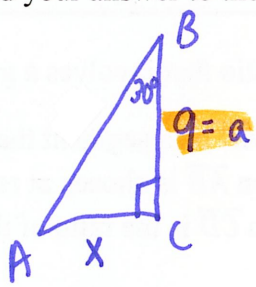
Example #7: Find the area of the inscribed hexagon. Round your answer to the nearest thousandth.



Central ∠

$\frac{360}{6} = 60$

$\frac{60}{2} = 30°$



$X = \frac{9}{\sqrt{3}}$

$X = 5.20$

$S = 2(5.20)$

$S = 10.4$

$P = 6(10.4)$

$P = 62.4$

$A = \frac{a \cdot P}{2}$

$A = \frac{9(62.4)}{2}$

$A = 280.8 u^2$

Finding Lengths in a Regular N-gon

To find the area of a regular n-gon with <u>radius *r*</u>, you may need to first find the <u>apothem *a*</u> or the <u>side length *s*.</u>

You can use…	…when you know *n* and…	Example(s) to Reference
Pythagorean Thm $a^2 + b^2 = c^2$	two measures "r" and "a" or "r" and "s"	ex 5
Special Δ's 30-60-90 or 45-45-90 (hexagon)	any one measure r, a, or s	ex 7
Trig ratios SOH-CAH-TOA	any one measure r, a, or s AND find your central ∠ (1st) $\frac{360}{n}$	ex 6

Chapter 11.7: Use Geometric Probability

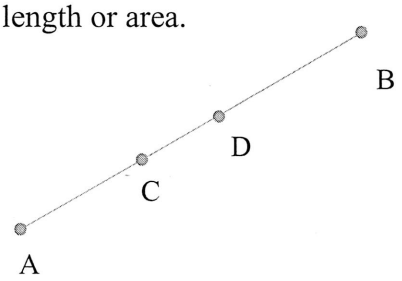
Probability: the likelihood that an event will occur.

Probability = _____

$P = 0$ $P = 0.25$ $P = 0.50$ $P = 0.75$ $P = 1$

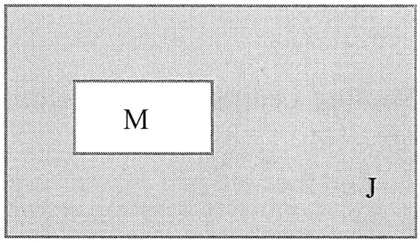
Geometric Probability: A ratio that involves a geometric measure such as length or area.

Probability and Length: Let $\overline{AB}$ be a segment that contains the segment $\overline{CD}$. If a point K on $\overline{AB}$ is chosen at random, then the probability that it is on $\overline{CD}$ is the ratio of the length of $\overline{CD}$ to the length of $\overline{AB}$.



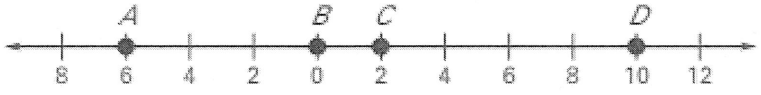
P(K is on $\overline{CD}$) = _____

Probability and Area: Let J be a region that contains region M. If a point K in J is chosen at random, then the probability that it is in region M is that ratio of the area of M to the area of J.



P(K is in region M) = _____

Example #1: Find the probability that a point chosen at random on $\overline{AD}$ is on the given line segment. Express your answer as a fraction, a decimal and a percent.



a) $\overline{AB}$ b) $\overline{BC}$ c) $\overline{AC}$ d) $\overline{BD}$