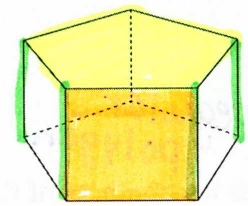
Chapter 12.2: Surface Area of Prisms and Cylinders

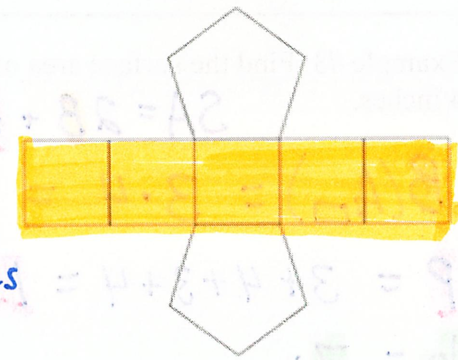
Prism: Polyhedron with 2 congruent faces (bases) that lie in parallel planes.



Lateral Faces: Parallelograms formed by connecting the corresponding vertices of the bases

Lateral Edge: Segments connecting the vertices of the base.

Net: two dimensional representations of the faces of a polyhedron.



"wrapping paper"

Surface Area of a polyhedron, is the sum of the area of all its faces.

Lateral Area of a polyhedron is the sum of the area of the lateral faces.

Total Surface Area - Area of Bases

Example #1: Find the surface area of a rectangular prism with height 2cm, length 5 cm and width 6 cm.

5 ① 6 $A = 30cm^2 \times 2 \rightarrow 60cm^2$

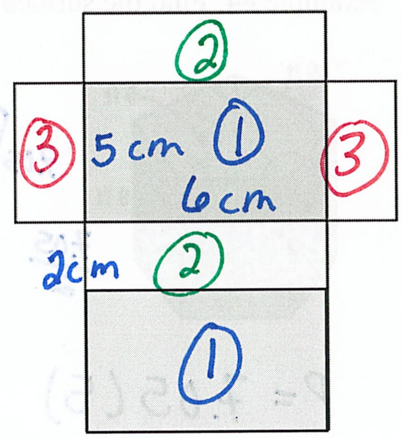
+

2 ② 6 $A = 12 cm^2 \times 2 \rightarrow 24 cm^2$

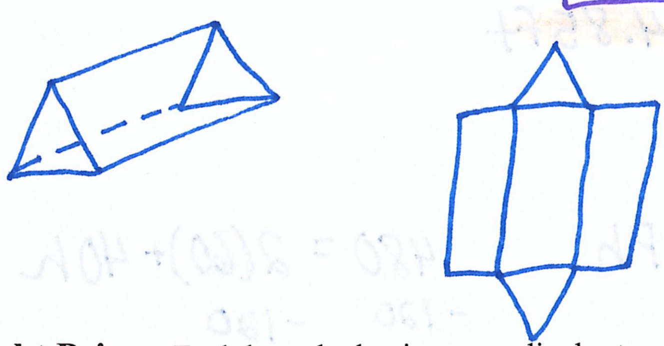
+

2 ③ 5 $A = 10 cm^2 \times 2 \rightarrow 20 cm^2$

$\boxed{SA = 104\ cm^2}$

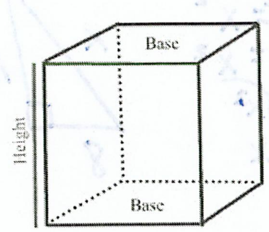


Example #2: Draw a net of a triangular prism.

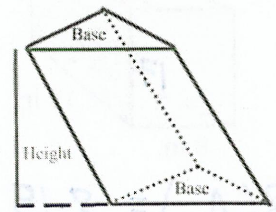


Right Prism: Each lateral edge is perpendicular to both bases.

Oblique Prism: Each lateral edge is not perpendicular to the bases.



right



Oblique

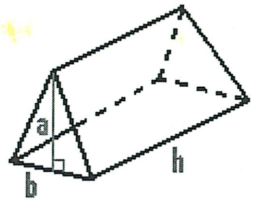
Surface Area of a Right Prism:

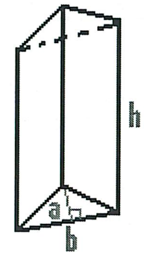
reg. polygon base $A = \frac{aP}{2}$ $\quad \frac{\cancel{2}}{1}\left(\frac{aP}{\cancel{2}}\right) = aP$

The surface area S of a right prism is

$$SA = 2B + Ph$$

* regular polygon $SA = aP + Ph$





Where *a* is the apothem of the base, *B* is the area of a base, varies depending on the shape of Base
P is the perimeter of a base, and *h* is the height.

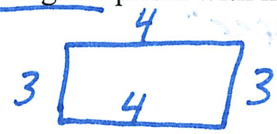
Example #3: Find the surface area of a right rectangular prism with height 7 inches, length 3 inches and width 4 inches.

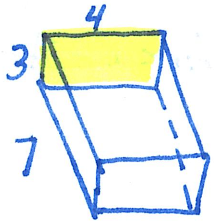
$SA = 2B + Ph$

$B(A_{\square}) = 3 \cdot 4 = 12$ in

$P = 3+4+3+4 = 14$ in

$h = 7$ in



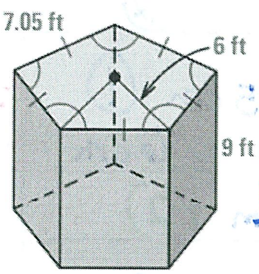


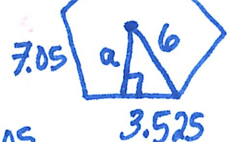
$SA = 2(12) + 14(7)$

$= 24 + 98$

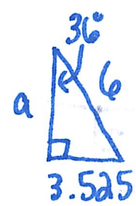
$\boxed{SA = 122 \text{ in}^2}$

Example #4: Find the surface area of the right pentagonal prism.





$\frac{7.05}{2}$



trig

reg. polygon

$a^2 + b^2 = c^2$

or

trig

$a^2 + 3.525^2 = 6^2$

$\quad -3.525^2 \;\; -3.525^2$

$\sqrt{a^2} = \sqrt{23.57}$

$a \approx 4.85$ ft

$P = 7.05(5)$

$P = 35.25$ ft

$h = 9$ ft

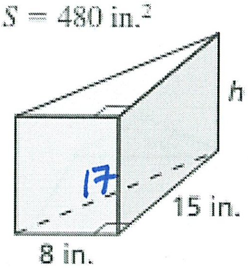
$SA = aP + Ph$

$= 4.85(35.25) + 35.25(9)$

$\boxed{SA = 488.21 \text{ ft}^2}$

Example #5: Find the height of the right prism

$S = 480 \text{ in.}^2$

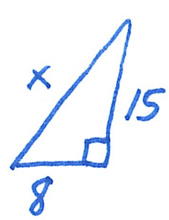


$8^2 + 15^2 = x^2$

$64 + 225 = x^2$

$\sqrt{289} = \sqrt{x^2}$

$17 = x$



$SA = 2B + Ph$

$480 = 2(60) + 40h$

$-120 \quad -120$

$\frac{360}{40} = \frac{40h}{40}$

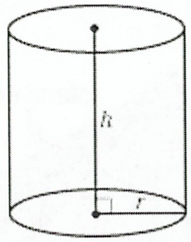
$\boxed{9 \text{ in} = h}$

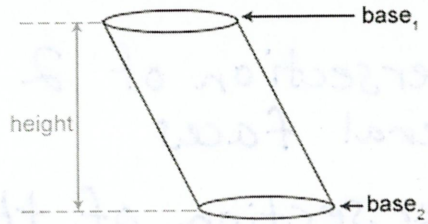
$B(A_{\triangle}) = \frac{8 \cdot 15}{2} = 60$

$P = 8 + 15 + 17 = 40$

Cylinder: A solid with congruent, circular, parallel bases.

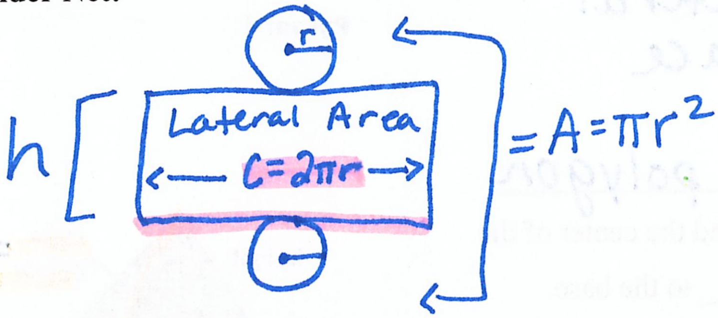
Right Cylinder: Segment joining the centers of the bases is perpendicular to the bases.





Oblique cylinder

Cylinder Net:



Surface Area of a Right Cylinder:

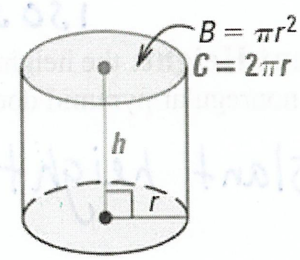
* only need 2 things:
1) radius
2) height

The surface area S of a right cylinder is

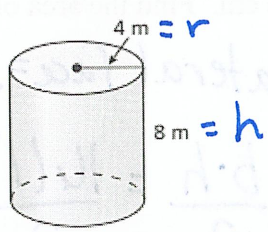
$$SA = 2B + Ch \quad \text{or}$$

$$* \; SA = 2(\pi r^2) + (2\pi r)h$$

Where B is the area of a base, C is the circumference of a base, r is the radius of a base and h is the height.



Example #5: Find the surface area of the right cylinder.

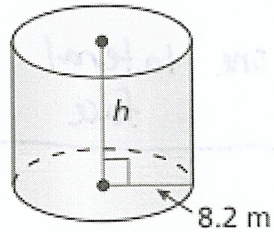


$$SA = 2(\pi r^2) + (2\pi r)h$$
$$= 2\pi 4^2 + 2\pi 4(8)$$
$$= 32\pi + 64\pi$$
$$= 96\pi$$

$$\boxed{SA \approx 301.59 m^2}$$

Example #6: Find the height of the right cylinder.

$S = 1097$ m^2



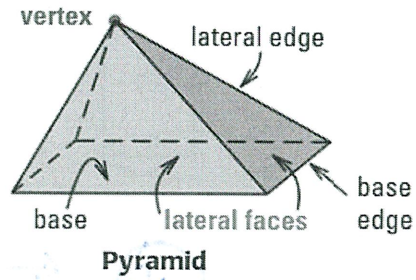
$$SA = 2\pi r^2 + 2\pi r h$$
$$1097 = 2\pi(8.2)^2 + 2\pi 8.2(h)$$
$$1097 = 134.48\pi + 16.4\pi h$$
$$-134.48\pi \quad -134.48\pi$$
$$\frac{674.52}{(16.4\pi)} = \frac{16.4\pi h}{(16.4\pi)}$$

$$\boxed{h \approx 13.09 m}$$

Pyramid: a polyhedron in which the base is a polygon and the lateral faces are triangles with a common vertex.

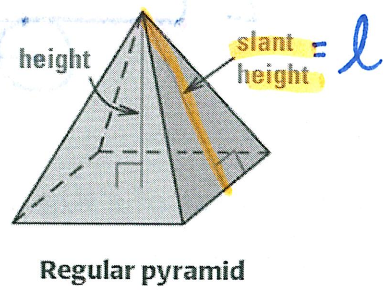
Lateral Edge: intersection of 2 lateral faces

Base Edge: intersection of the base and a lateral face



Regular Pyramid: has a regular polygon for a base and the segment joining the vertex and the center of the base is perpendicular to the base.

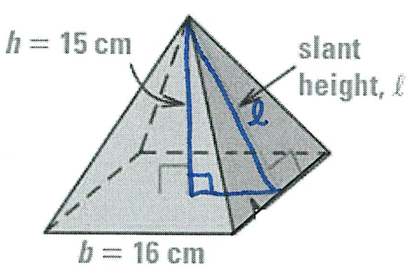
Lateral Faces are all congruent isosceles triangles

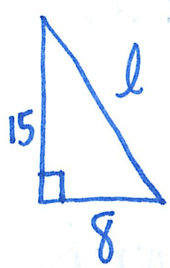


Slant Height: the height of a lateral face of the regular pyramid (A nonregular pyramid does not have aslant height)

slant height – ℓ – height of the lateral Δ

Example #1: A regular square pyramid has height of 15 cm and a base edge length of 16 cm. Find the area of each lateral face of the pyramid.





$15^2 + 8^2 = \ell^2$

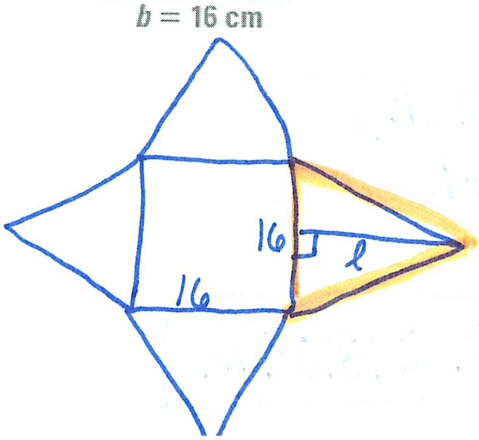
$\sqrt{225 + 64} = \sqrt{\ell^2}$

$17 = \ell$

1 lateral face = 1 Δ

$A = \frac{b \cdot h}{2} = \frac{16(17)}{2}$

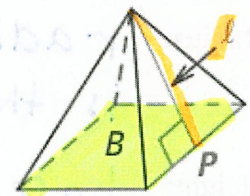
$\boxed{A = 136\ cm^2 \text{ one lateral face}}$



Surface Area of a Regular Pyramid (Theorem 12.4):

The surface area S of a regular pyramid is

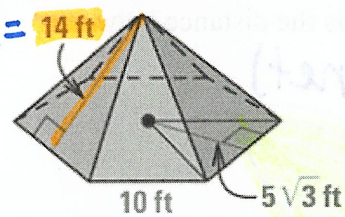
$$SA = B + \frac{1}{2}P\ell$$



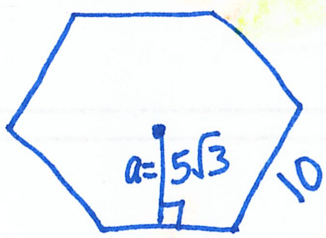
where *B* is the area of the Base, *P* is the perimeter of the base, and *l* is the slant height.

Example #2: Find the surface area of the regular hexagonal pyramid.

$\ell =$



$P = 6(10) = 60\text{ ft}$



$$B(A_{Hex}) = \frac{aP}{2}$$

$$= \frac{5\sqrt{3}(60)}{2}$$

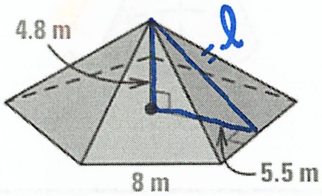
$$B \approx 259.8\text{ ft}^2$$

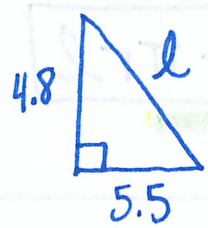
$$SA = B + \frac{1}{2}P\ell$$

$$= 259.8 + \frac{1}{2}(60)(14)$$

$$\boxed{SA = 679.8\text{ ft}^2}$$

Example #3: Find the surface area of the regular pentagonal pyramid shown





$$\sqrt{4.8^2 + 5.5^2} = \sqrt{\ell^2}$$

$$7.3 \approx \ell$$

$P = 5(8) = 40\text{ ft}$

$a = 5.5$

$$B = \frac{aP}{2}$$

$$= \frac{5.5(40)}{2}$$

$$B = 110\text{ m}^2$$

$$SA = B + \frac{1}{2}P\ell$$

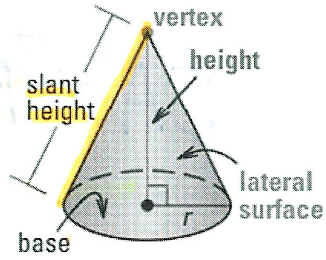
$$= 110 + \frac{1}{2}(40)(7.3)$$

$$\boxed{SA = 256\text{ m}^2}$$

Cone: a solid with a circular base and a vertex that is not in the same plane as the base.

Radius: radius of the base is the radius of the cone

Height: base to the vertex (perpendicular) right cone

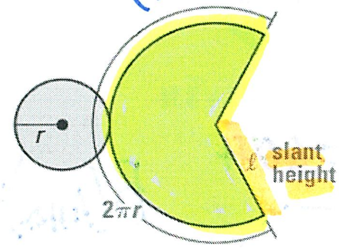


Right cone

In a right cone, the segment joining the vertex and the center of the base is perpendicular to the base, and the slant height is the distance between the vertex and a point of the base edge.

(net)

Lateral Surface: consists of all segments that connect the vertex to the base (no base)

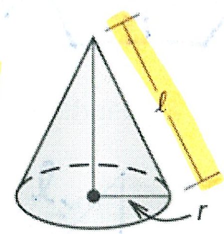


Surface Area of a Right Cone (Theorem 12.5):

The surface area S of a right cone is

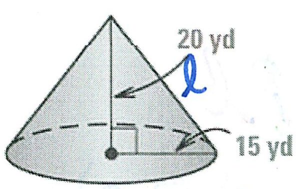
$$SA = B + \frac{1}{2}C\ell \rightarrow SA = \pi r^2 + \frac{1}{2}(2\pi r)\ell$$

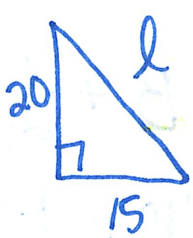
$$* \boxed{SA = \pi r^2 + \pi r \ell}$$



where B is the area of the Base, C is the circumference of the base, r is the radius of the base, and l is the slant height.

Example #4: Find the lateral area of the right cone.





$$\sqrt{20^2 + 15^2} = \sqrt{\ell^2}$$

$$25 = \ell$$

$$r = 15$$

$$SA = \cancel{B} + \underbrace{\frac{1}{2}P\ell}_{\text{lateral area}}$$

Perimeter = Circumference in a circle

$$\text{Lateral Area} = \frac{1}{2}C\ell$$

$$LA = \frac{1}{\cancel{2}}(\cancel{2}\pi r)\ell$$

$$= \pi r \ell$$

$$= \pi\, 15(25)$$

$$\boxed{LA \approx 1178.1\ yd^2}$$