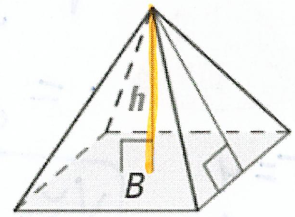
Chapter 12.5: Volume of Pyramids and Cones

Volume of a Pyramid (Theorem 12.9):

The volume V of a pyramid is

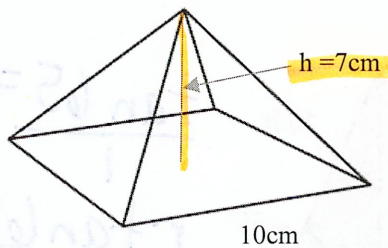
$$V = \frac{1}{3}Bh$$

where B is the area of the base and h is the height.



Example #1: Find the volume of the pyramid with the regular base. Round answers to the nearest hundredth.

a.)

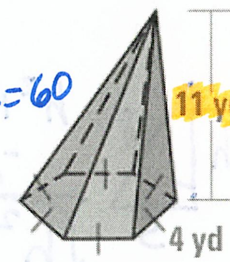


$V = \frac{1}{3}Bh \rightarrow \frac{1}{3}(100)(7)$

$B = 10^2 = 100$

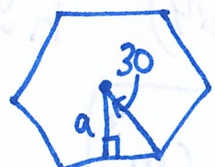
$\boxed{V \approx 233.33\ cm^3}$

b.)



$\frac{360}{6} = 60$

$\frac{60}{2}$ 30



$h = 11$

$B = \frac{aP}{2}$

$P = 4(6) = 24$

$\frac{\tan 30}{1} = \frac{2}{a}$

$\frac{a \tan 30}{\tan 30} = \frac{2}{\tan 30}$

$a \approx 3.46$

$B = \frac{3.46(24)}{2}$

$B = \frac{83.04}{2}$

$B = 41.52$

$V = \frac{1}{3}Bh$

$V = \frac{1}{3}(41.52)(11)$

$V \approx \frac{456.72}{3}$ yd

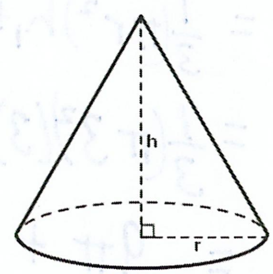
$\boxed{V \approx 152.24\ yd}$

Volume of a Cone (Theorem 12.10):

The volume V of a cone is

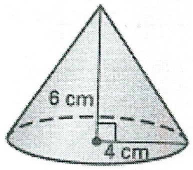
$$V = \frac{1}{3}Bh \rightarrow V = \frac{1}{3}(\pi r^2)h$$

where B is the area of the base, h is the height, and r is the radius of the base.



Example #2: Find the volume of each cone. Round answers to the nearest hundredth.

a.)



$V = \frac{1}{3}(\pi 4^2)(6)$

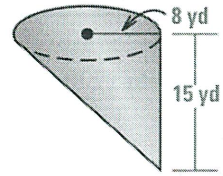
$= 32\pi$

$r = 4$

$h = 6$

$\boxed{V \approx 100.53 \text{ cm}^3}$

b.)

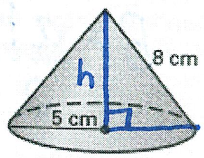


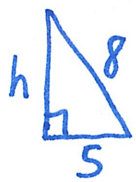
$V = \frac{1}{3}(\pi 8^2)(15)$

$= 320\pi$

$\boxed{V \approx 1005.31 \text{ yd}^3}$

c.)





$h^2 + 5^2 = 8^2$

$\quad -25 \quad -25$

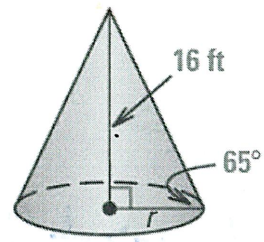
$\sqrt{h^2} = \sqrt{39}$

$h \approx 6.24$

$V = \frac{1}{3}(\pi 5^2)(6.24)$

$\boxed{V \approx 163.36 \text{ cm}^3}$

d.)



$\frac{\tan 65}{1} = \frac{16}{r}$

$\frac{r \tan 65}{\tan 65} = \frac{16}{\tan 65}$

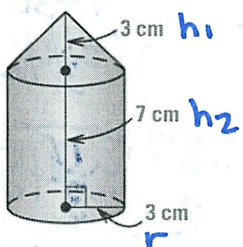
$r \approx 7.46$

$V = \frac{1}{3}(\pi 7.46^2)(16)$

$\boxed{V \approx 932.45 \text{ ft}^3}$

Example #3: Find the volume of the solid shown. Round answers to the nearest hundredth.

a.)



$V_{tot} = V_{cone} + V_{cyl}$

$= \frac{1}{3}(\pi r^2)h_1 + (\pi r^2)h_2$

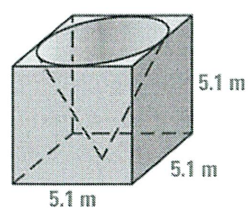
$= \frac{1}{3}(\pi 3^2)(3) + (\pi 3^2)(7)$

$= 9\pi + 63\pi$

$= 72\pi$

$\boxed{V \approx 226.19 \text{ cm}^3}$

b.)



$r = \frac{5.1}{2} = 2.55$

$h = 5.1$

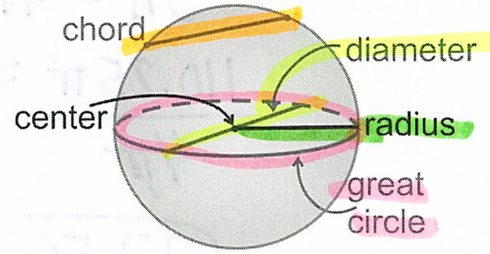
$V_{tot} = V_{cube} - V_{cone}$

$= s^3 - \frac{1}{3}(\pi r^2)h$

$= 5.1^3 - \frac{1}{3}\pi(2.55)^2(5.1)$

$\boxed{V = 97.92 \text{ m}^3}$

A sphere is the set of all points in space equidistant from a given point.



Center of a Sphere: the given point from which all points on the sphere is equidistant.

Radius of a Sphere: a segment from the center to any point on the sphere

Chord of a Sphere: a segment whose endpoints are on the sphere.

Diameter of a Sphere: a chord that contains the center of the sphere.

Great Circle: the intersection of a sphere and plane that contains the center of the sphere.

Hemisphere: one of the congruent halves of a sphere.

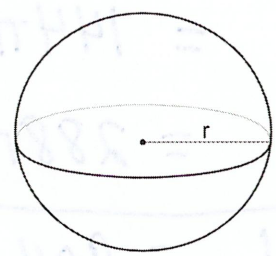
Surface Area of a Sphere (Theorem 12.11):

The surface area S of a sphere is

$$SA = 4\pi r^2$$

where r is the radius of the sphere.

* r^2 Area is squared 2D!



Volume of a Sphere (Theorem 12.12):

The volume V of a sphere is

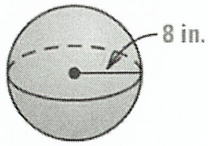
$$V = \frac{4}{3}\pi r^3$$

* r^3 volume is cubed 3D!

where r is the radius of the sphere.

Example #1: Find the surface area and volume of the sphere. Round answers to the nearest hundredth.

a.)



$SA = 4\pi r^2$

$= 4\pi 8^2$

$SA = 256\pi$

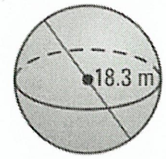
$\boxed{SA \approx 804.25 in^2}$

$V = \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi 8^3$

$\boxed{V \approx 2144.66 in^3}$

b.)



$r = 9.15$

$SA = 4\pi 9.15^2$

$\boxed{SA \approx 1052.09 m^2}$

$V = \frac{4}{3}\pi 9.15^3$

$\boxed{V \approx 3208.87 m^3}$

Example #2: The surface area of a sphere is 110.25π ft^2. Find the diameter of the sphere. Round answers to the nearest hundredth.

$$SA = 4\pi r^2$$

$$\frac{110.25\cancel{\pi}}{4\cancel{\pi}} = \frac{4\pi r^2}{4\pi}$$

$$\sqrt{27.56} = \sqrt{r^2} \qquad r = 5.25$$

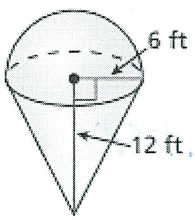
$$d = 2r$$

$$d = 2(5.25)$$

$$\boxed{d = 10.5 \text{ ft}}$$

Example #3: Find the volume of the composite solid. Round answers to the nearest hundredth.

a.)



$r = 6$

$h = 12$

$$V_{tot} = V_{cone} + \frac{1}{2} V_{sphere}$$

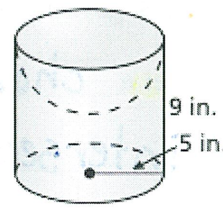
$$= \frac{1}{3}(\pi 6^2)12 + \frac{1}{\cancel{2}}\left(\frac{\overset{2}{\cancel{4}}}{3}\pi 6^3\right)$$

$$= 144\pi + 144\pi$$

$$= 288\pi$$

$$\boxed{V_{tot} = 904.78 \text{ ft}^3}$$

b.)



$r = 5$

$h = 9$

$$V_{tot} = V_{cyl} - V_{hemi.}$$

$$= (\pi 5^2)9 - \frac{1}{\cancel{2}}\left(\frac{\overset{2}{\cancel{4}}}{3}\pi 5^3\right)$$

$$= 225\pi - \frac{250\pi}{3}$$

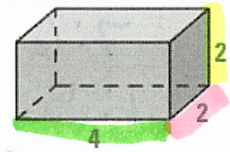
$$\boxed{V_{tot} = 445.06 \text{ in}^3}$$

Chapter 12.7: Explore Similar Solids

Similar Solids: Two Solids of same type with equal ratios of corresponding linear measures.

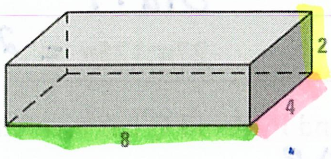
Scale Factor: common ratio to go from one solid to the other.

$a:b$



Example #1: Tell whether the given right rectangular prism is similar to the right rectangular prisms shown below.

a.)



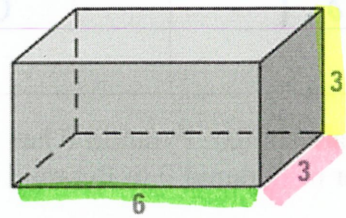
NO

$\frac{2}{2} = 1$ ✗

$\frac{2}{4} = \frac{1}{2}$ ✓

$\frac{4}{8} = \frac{1}{2}$ ✓

b.)



similar

$\frac{2}{3}$ ✓

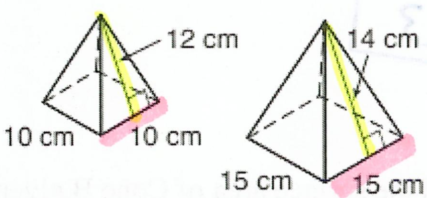
$\frac{2}{3}$ ✓

$\frac{4}{6} = \frac{2}{3}$ ✓

same

Example #2: Tell whether the pair of solids is similar.

a.)



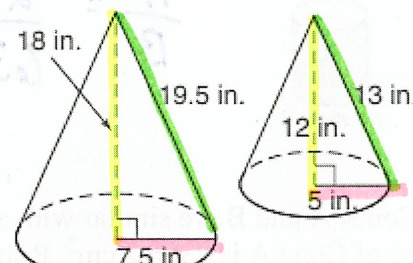
$\frac{12}{14} = \frac{6}{7}$

$\frac{10}{15} = \frac{2}{3}$

$\frac{6}{7} \neq \frac{2}{3}$

Not similar

b.)



$\frac{18}{12} = \frac{3}{2}$

$\frac{7.5}{5} = \frac{3}{2}$

$\frac{19.5}{13} = \frac{3}{2}$

yes similar

Similar Solids Theorem (Theorem 12.13):

If two similar solids have a scale factor of $a:b$, (side length, radii, altitudes perimeter, etc)

then corresponding areas have a ratio of $a^2:b^2$,

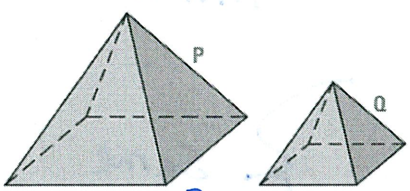
and corresponding volumes have a ratio of $a^3:b^3$.

Example #2: Fill in the chart

$a^2 : b^2$ $\qquad$ $a^3 : b^3$

Ratio of perimeter/corresponding lengths (scale factor)	Ratio of Areas (surface area)	Ratio of Volumes
3:4	9:16	27:64
7:6	49:36	343:216
1:5	1:25	1:125
24:3= 8:1	64:1	512:1
3:5	9:25	$27\pi:125\pi$ = 27:125

Example #3: The pyramids are similar. Pyramid P has a volume of 1000 in^3 and Pyramid Q has a volume of 216 in^3. Find the scale factor of Pyramid P to Pyramid Q.



V= 1000 in^3 $\qquad$ V= 216 in^3

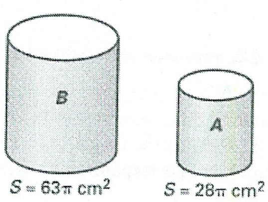
R.O.V $\qquad$ R.O.P

$$\frac{P}{Q} = \frac{1000}{216} = \frac{\sqrt[3]{1000}}{\sqrt[3]{216}} = \frac{10}{6} = \boxed{\frac{5}{3}}$$

or

$$\frac{125}{27} = \frac{\sqrt[3]{125}}{\sqrt[3]{27}}$$

Example #4: The two cylinders are similar. Find the scale factor of Cylinder A to Cylinder B.



$$\frac{A}{B} = \frac{28\pi}{63\pi} = \frac{\sqrt{4}}{\sqrt{9}} = \boxed{\frac{2}{3}}$$

Example #5: Cones A and B are similar with a scale factor of 5:2. Find the surface area of Cone B given that the surface area of Cone A is 2356.2 cm^2. Round your answer to the nearest hundredth.

$$\frac{A}{B} = \frac{5}{2}$$

S.F $\qquad$ R.O.A

$$\frac{A}{B} \quad \frac{5^2}{2^2} = \frac{25}{4}$$

$$\frac{25}{4} = \frac{2356.2}{B}$$

$$\frac{25B}{25} = \frac{9424.8}{25}$$

$$\boxed{SA(B) = 376.99 \text{ cm}^2}$$

Find the volume of Cone B given that the volume of Cone A is 7450.9 cm^3. Round your answer to the nearest hundredth.

R.O.V

$$\frac{5^3}{2^3} = \frac{125}{8}$$

$$\frac{125}{8} = \frac{7450.9}{B}$$

$$\frac{125B}{125} = \frac{59607.2}{125}$$

$$\boxed{V(B) = 476.86 \text{ cm}^3}$$