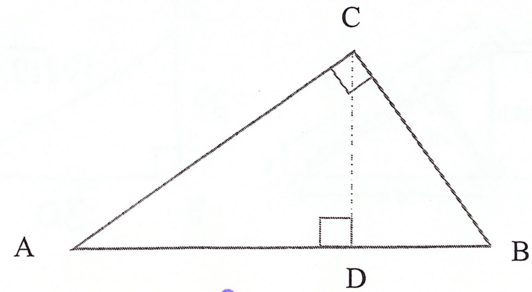
Chapter 7.3: Use Similar Right Triangles

Altitude Similarity Theorem **(Theorem 7.5):**

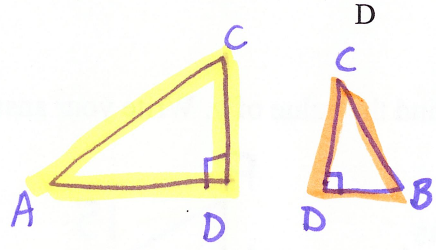
If the altitude is drawn to the hypotenuse of a **right triangle**, then the 2 triangles formed are similar to the original triangle and each other.



$\triangle ABC \sim \triangle ACD$

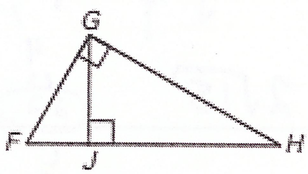
$\triangle ABC \sim \triangle CBD$

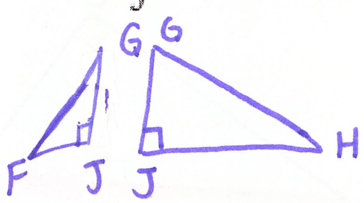
$\triangle ACD \sim \triangle CBD$



Example #1: Identify the similar triangles in the diagram

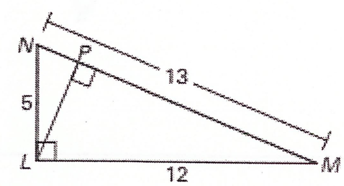
a.)

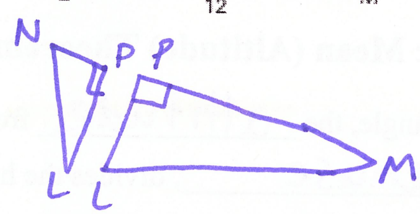




$\triangle FGH \sim \triangle FJG \sim \triangle GJH$

b.)

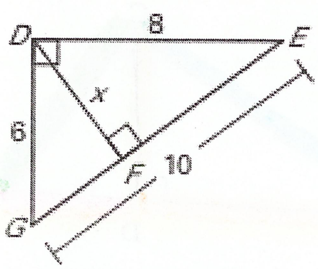




$\triangle NLM \sim \triangle NPL \sim \triangle LPM$

Example #2: Identify the similar triangles. Then find *x*

1



$\triangle GDE \sim \triangle GFD \sim \triangle DFE$

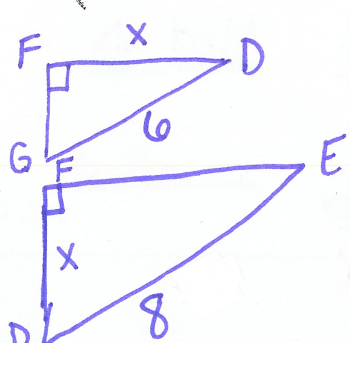
$$\frac{(\text{hyp } 3)}{(\text{hyp } 1)} \quad \frac{8}{10} = \frac{x}{6}$$

$$\frac{10x}{10} = \frac{48}{10}$$

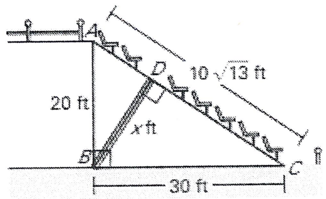
$$\boxed{x = 4.8}$$

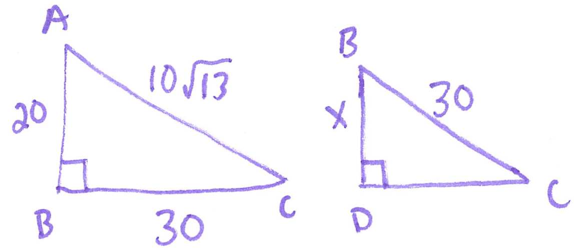
2

3



Example #3: A cross section of a group of seats at a stadium shows a drainage pipe $\overline{BD}$ that leads from the seats to the inside of the stadium. What is the length of the pipe?





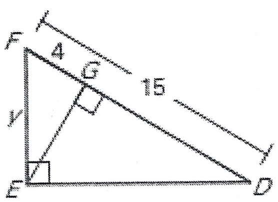
$$\frac{20}{x} = \frac{10\sqrt{13}}{30}$$

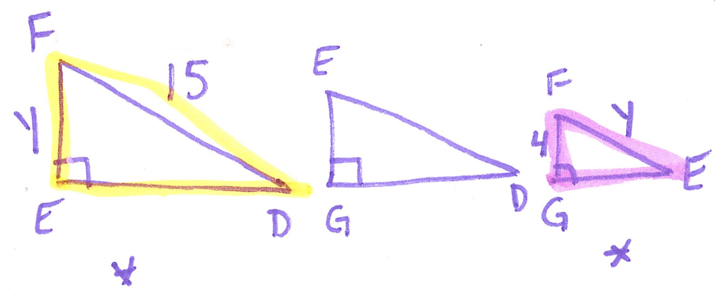
$$\frac{600}{10} = \frac{10x\sqrt{13}}{10}$$

$$\frac{60}{\sqrt{13}} = \frac{x\sqrt{13}}{\sqrt{13}}$$

$$\boxed{x \approx 16.6\text{ft}}$$

Example #4: Find the value of y. Write your answer in simplest radical form





$$\frac{15}{y} = \frac{y}{4}$$

$$\sqrt{y^2} = \sqrt{60}$$

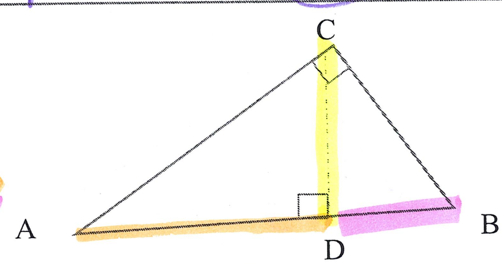
60 → 4, 15; 4 → 2, 2

$$y = 2\sqrt{15}$$

Geometric Mean (Altitude) Theorem (Theorem 7.6):

In a right triangle, the altitude from the right angles to the hypotenuse, divides the hypotenuse into two segments.

The length of the **altitude** is the geometric mean of the lengths of the two segments.

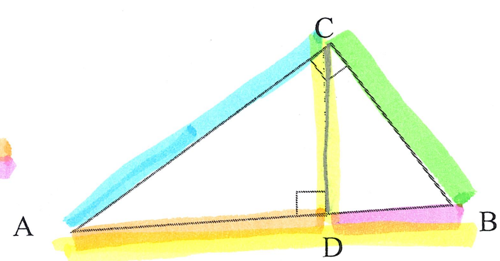


$$\frac{AD}{CD} = \frac{CD}{BD}$$

Geometric Mean (Leg) Theorem (Theorem 7.7):

In a right triangle, the altitude from the right angles to the hypotenuse, divides the hypotenuse into two segments

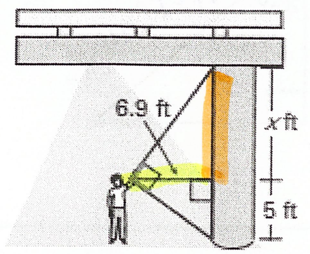
The length of each leg of the right triangle is the geometric mean of the lengths of the Hypotenuse and the segment of the hypotenuse that is adjacent to the leg



① $\frac{AB}{AC} = \frac{AC}{AD}$ Med Δ

② $\frac{AB}{BC} = \frac{BC}{BD}$ small Δ

Example #5: To find the clearance under an overpass, you need to find the height of a concrete support beam. You use a cardboard square to line up the top and bottom of the beam. Your friend measures the vertical distance from the ground to your eye and the distance from you to the beam. Approximate (dec) the height of the beam.



$$\frac{x}{6.9} = \frac{6.9}{5}$$

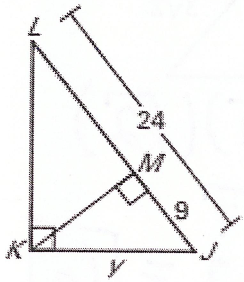
$$\frac{5x}{5} = \frac{6.9^2}{5}$$

$$x \approx 9.5$$

height = $x + 5$

$9.5 + 5 = \boxed{14.5\text{ ft}}$

Example #6: Find the value of y. Write your answer in simplest radical form.



L, 24, K, y, J; y, 9

$$\frac{24}{y} = \frac{y}{9}$$

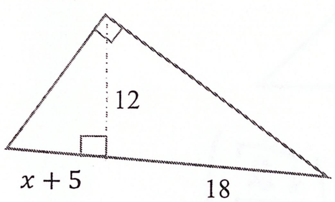
$$\sqrt{y^2} = \sqrt{216}$$

216 → 4, 54; 4 → 2 2; 54 → 6 9; 6 → 2 3; 9 → 3 3

$2 \cdot 3\sqrt{2 \cdot 3}$

$\boxed{6\sqrt{6}}$

Example # 7: Find the value of x.



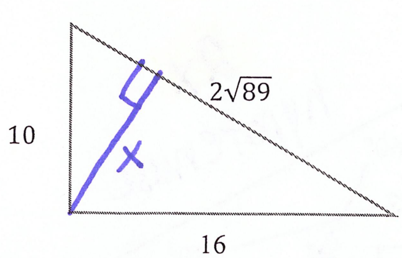
$$\frac{x+5}{12} = \frac{12}{18}$$

$$\frac{18(x+5)}{18} = \frac{144}{18}$$

$$x + 5 = 8$$
$$-5 \quad -5$$

$\boxed{x = 3}$

Example # 8: Tell whether the triangle is a right triangle. If so, find the length of the altitude to the hypotenuse. Round decimal answer to the nearest hundredth.



$$10^2 + 16^2 \stackrel{?}{=} (2\sqrt{89})^2$$

$$100 + 256 = 356$$

$$356 = 356 \checkmark$$

10, $2\sqrt{89}$, 16; x, 16

$$\frac{10}{x} = \frac{2\sqrt{89}}{16}$$

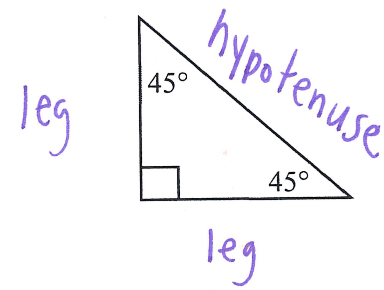
$$\frac{160}{(2\sqrt{89})} = \frac{\cancel{2}x\cancel{\sqrt{89}}}{(2\sqrt{89})}$$

$\boxed{8.48 \approx x}$

Hw: 7.3 wksht

45° – 45° – 90° Triangle Theorem (Theorem 7.8):

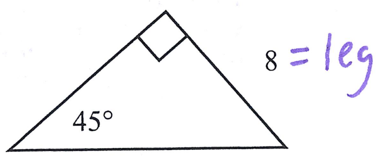
In a 45° – 45° – 90° triangle, the hypotenuse is $\sqrt{2}$ times as long as each leg.



Hypotenuse = $\text{leg} \cdot \sqrt{2}$ **Leg** = $\frac{\text{hyp}}{\sqrt{2}}$

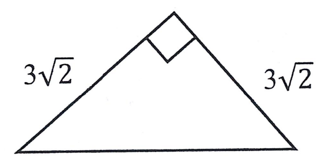
Example #1: Find the length of the hypotenuse.

a.



hyp = $8\sqrt{2}$

b.



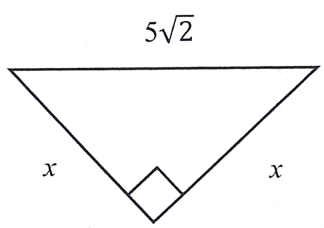
hyp = $(3\sqrt{2})(\sqrt{2})$

hyp = $3\sqrt{4}$

hyp = $3 \cdot 2 = \boxed{6}$

Example #2: Find the lengths of the legs in the triangle.

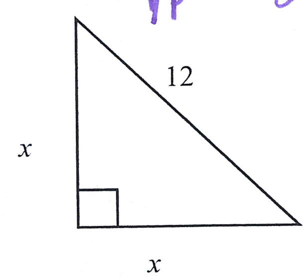
a.



$x = \frac{5\sqrt{2}}{\sqrt{2}}$

$\boxed{x = 5}$

b.

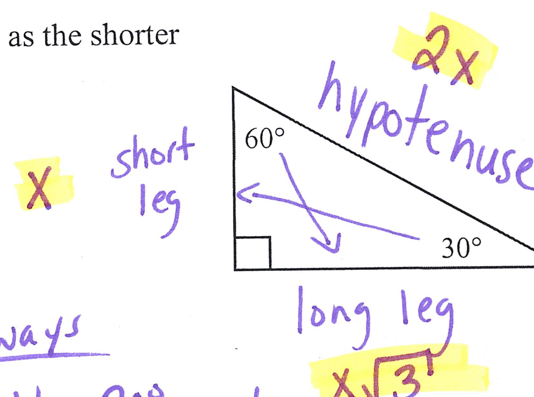


$x = \frac{12}{\sqrt{2}} \cdot \left(\frac{\sqrt{2}}{\sqrt{2}}\right)$

$x = \frac{12\sqrt{2}}{2} = \boxed{6\sqrt{2}}$

30° – 60° – 90° Triangle Theorem (Theorem 7.9):

In a 30° – 60° – 90° triangle, the hypotenuse is twice as long as the shorter leg, and the longer leg is $\sqrt{3}$ times as long as the shorter leg.

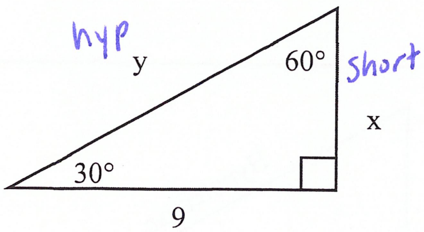


Hypotenuse = $2 \cdot \text{short leg}$

Longer Leg = $\text{Short leg} \cdot \sqrt{3}$

short leg is always across from the 30° angle

Example #3: Find the values of x and y. Write your answer in simplest radical form.



long = short $\sqrt{3}$

$\frac{9}{\sqrt{3}} = \frac{x\sqrt{3}}{\sqrt{3}}$

$\frac{9}{\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right) = x$

$\frac{9\sqrt{3}}{3} = x$

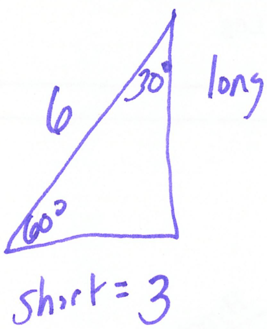
$\boxed{3\sqrt{3} = x}$

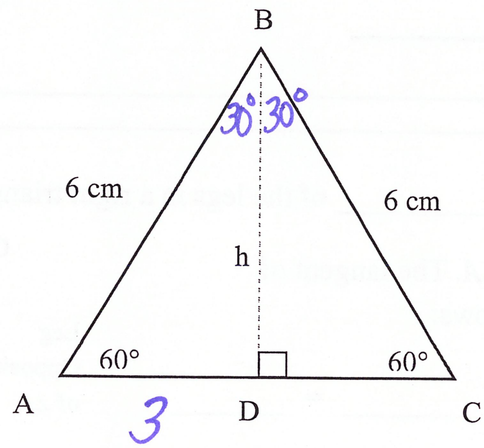
hyp = 2 short

$y = 2(3\sqrt{3})$

$\boxed{y = 6\sqrt{3}}$

Example #4: Find the height of the given triangle.





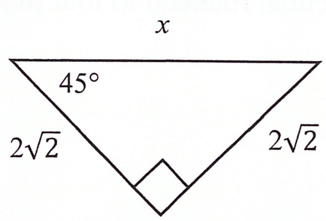
hyp = 2 · short

long = short $\sqrt{3}$

$\boxed{h = 3\sqrt{3} \text{ cm}}$

Check Point: Find the value of each variable.

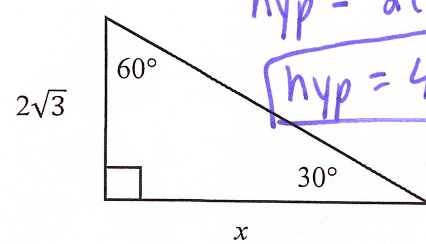
1.



$\text{hyp} = (2\sqrt{2})\sqrt{2}$

$\text{hyp} = 2 \cdot 2 = \boxed{4} = x$

2.



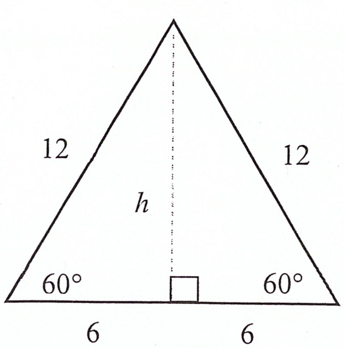
$\text{hyp} = 2(2\sqrt{3})$

$\boxed{\text{hyp} = 4\sqrt{3}}$

$\text{long} = 2(\sqrt{3})(\sqrt{3})$

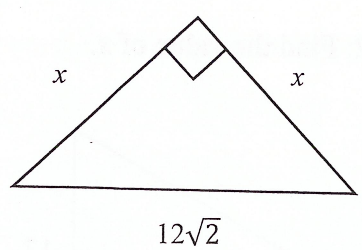
$\boxed{\text{long} = 6} = x$

3.



$\boxed{h = 6\sqrt{3}}$

4.



$\text{short} = \frac{\text{hyp}}{\sqrt{2}}$

$\text{short} = \frac{12\sqrt{2}}{\sqrt{2}}$

$\boxed{x = 12}$

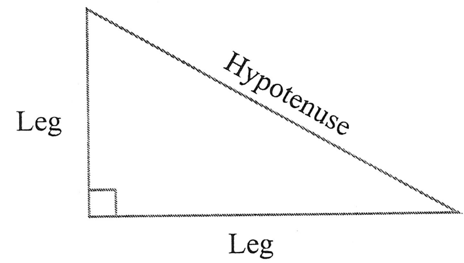
Chapter 7.5: Apply the Tangent Ratio

Definitions:

A **trigonometric ratio** is a ratio of the lengths of two sides in a right triangle. You will use trigonometric ratios to find…

1. ______________________________

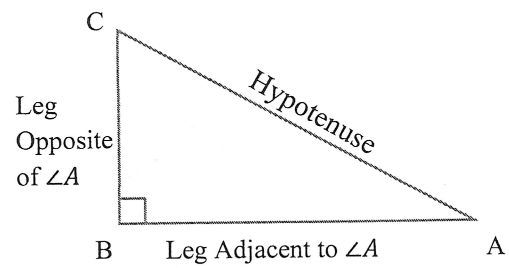
2. ______________________________



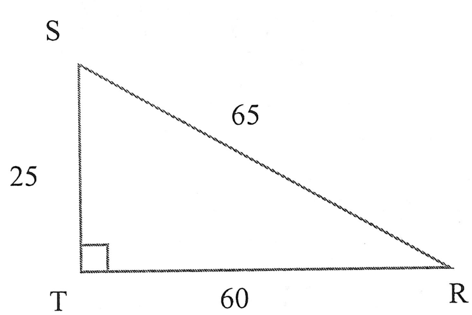
Tangent Ratio: the ratio of the ____________ of the legs in a right triangle.

Let $\triangle ABC$ be a right triangle with acute $\angle A$. The tangent of of $\angle A$ (written as tan A) is defined as follows:

tan A = ______________________________ = ________



Example #1: Find tan S and tan R. Write each answer as a fraction and as a decimal rounded to four places, if necessary.



Example #2: Find the value of x.