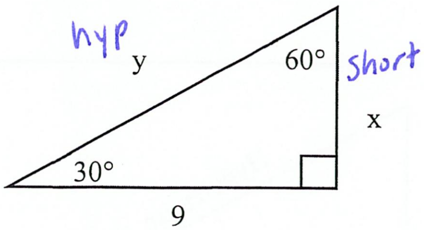
Example #3: Find the values of x and y. Write your answer in simplest radical form.



long = short $\sqrt{3}$

$\dfrac{9}{\sqrt{3}} = \dfrac{x\sqrt{3}}{\sqrt{3}}$

$\dfrac{9}{\sqrt{3}}\left(\dfrac{\sqrt{3}}{\sqrt{3}}\right) = x$

$\dfrac{9\sqrt{3}}{3} = x$

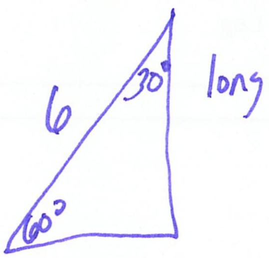
$\boxed{3\sqrt{3} = x}$

hyp = 2 short

$y = 2(3\sqrt{3})$

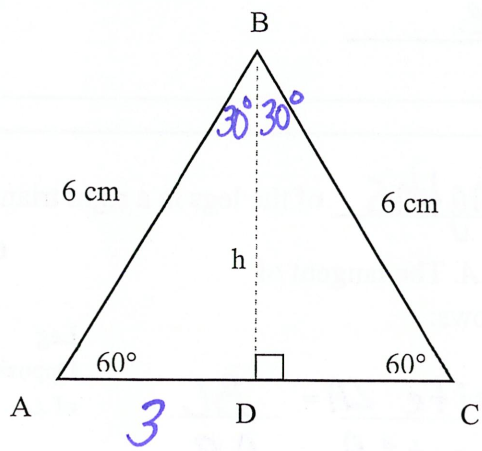
$\boxed{y = 6\sqrt{3}}$

Example #4: Find the height of the given triangle.



short = 3

$\boxed{h = 3\sqrt{3}\text{ cm}}$

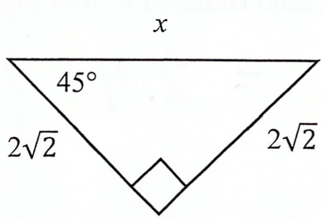


hyp = 2 · short

long = short $\sqrt{3}$

Check Point: Find the value of each variable.

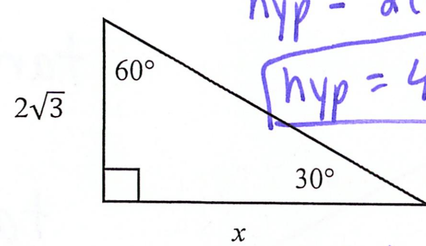
1.



$hyp = (2\sqrt{2})\sqrt{2}$

$hyp = 2 \cdot 2 = \boxed{4} = x$

2.



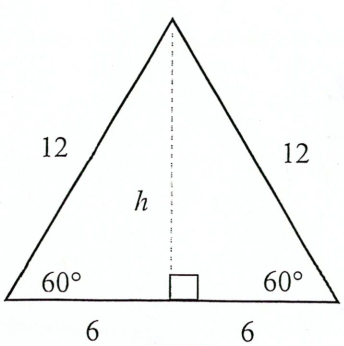
$hyp = 2(2\sqrt{3})$

$\boxed{hyp = 4\sqrt{3}}$

$long = 2(\sqrt{3})(\sqrt{3})$

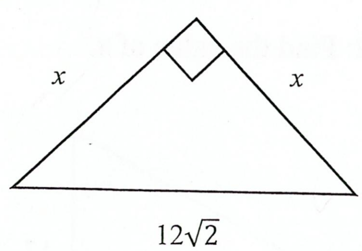
$\boxed{long = 6} = x$

3.



$\boxed{h = 6\sqrt{3}}$

4.



$short = \dfrac{hyp}{\sqrt{2}}$

$short = \dfrac{12\sqrt{2}}{\sqrt{2}}$

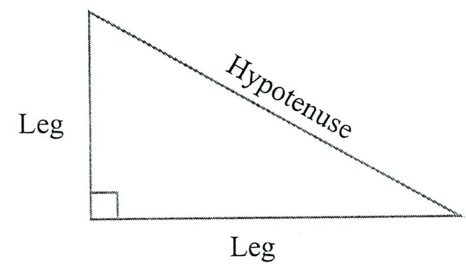
$\boxed{x = 12}$

Chapter 7.5: Apply the Tangent Ratio

Definitions:

A **trigonometric ratio** is a ratio of the lengths of two sides in a right triangle. You will use trigonometric ratios to find…

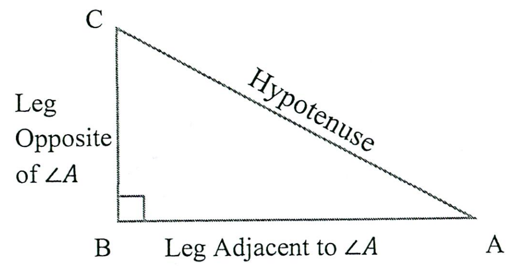
1. measure of a side
2. an acute angle



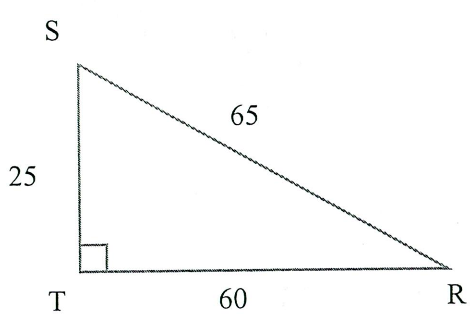
Tangent Ratio: the ratio of the lengths of the legs in a right triangle.

Let $\triangle ABC$ be a right triangle with acute $\angle A$. The tangent of of $\angle A$ (written as tan A) is defined as follows:

$$\tan A = \frac{\text{the length of leg opposite } \angle A}{\text{length of leg adjacent } \angle A} = \frac{BC}{AB}$$



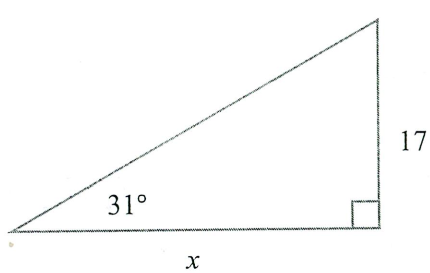
Example #1: Find tan S and tan R. Write each answer as a fraction and as a decimal rounded to four places, if necessary.



$$\tan S = \frac{60}{25} = \frac{12}{5} = 2.4$$

$$\tan R = \frac{25}{60} = \frac{5}{12} = .4167$$

Example #2: Find the value of x.



$$\tan 31° = \frac{17}{x}$$

$$\frac{\tan 31°}{1} = \frac{17}{x}$$

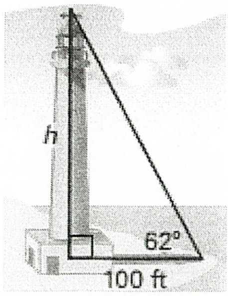
$$\frac{x \tan 31°}{\tan 31°} = \frac{17}{\tan 31}$$

$\boxed{x = 28.2928}$

or

$\boxed{28.3 = x}$

Example #3: Find the height h of the lighthouse to the nearest foot.

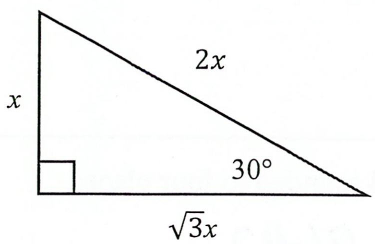


$100(\tan 62) = \left(\frac{h}{100}\right)100$

$100 \tan 62 = h$

$\boxed{188 \text{ ft} = h}$

Example #4: Use a **special right triangle** to find the tangent of a 30° angle.



$\tan 30° = \frac{x}{\sqrt{3}x}$

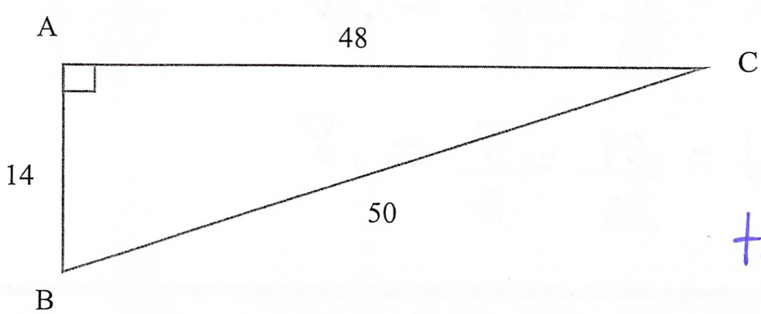
$\tan 30° = \frac{1}{\sqrt{3}} \cdot \left(\frac{\sqrt{3}}{\sqrt{3}}\right)$

$\tan 30° = \boxed{\frac{\sqrt{3}}{3}}$

Checkpoint:

#1: Find tan B and tan C.

Write each answer as a fraction and as a decimal rounded to four places.



$\tan B = \frac{48}{14} = \frac{24}{7}$

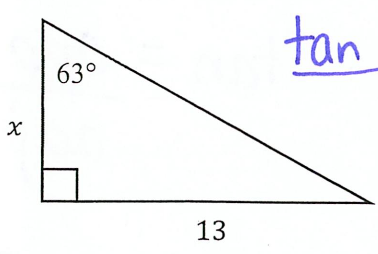
$\tan B = 3.4286$

$\tan C = \frac{14}{48} = \frac{7}{24}$

$\tan C = .2917$

Find the value of x. Round to the nearest tenth.

#2:

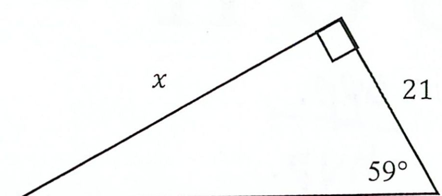


$\frac{\tan 63}{1} = \frac{13}{x}$

$\frac{x \tan 63}{\tan 63} = \frac{13}{\tan 63}$

$\boxed{x = 6.6}$

#3:



$21(\tan 59°) = \left(\frac{x}{\cancel{21}}\right)\cancel{21}$

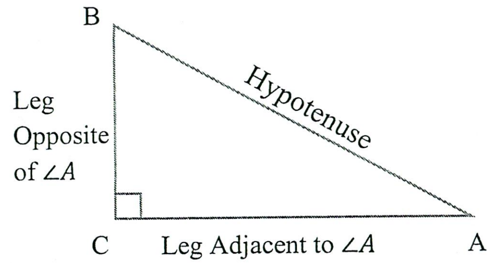
$\boxed{34.9 = x}$

Chapter 7.6: Apply the Sine and Cosine Ratios

Definitions:

Sine and Cosine Ratio:

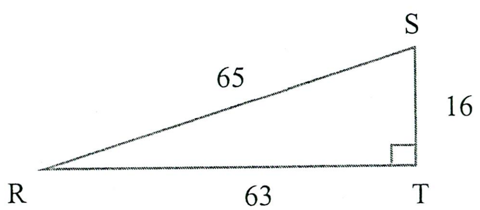
Let $\triangle ABC$ be a right triangle with acute $\angle A$.
The sine and cosine of $\angle A$ (written as sin A and cos A) are defined as follows:



$$\sin A = \frac{\text{length of leg opposite } \angle A}{\text{length of hypotenuse}} = \frac{BC}{BA}$$

$$\cos A = \frac{\text{length of leg adjacent } \angle A}{\text{length of hypotenuse}} = \frac{CA}{BA}$$

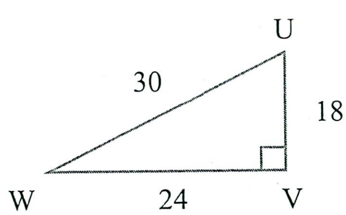
Example #1: Find sin S and sin R. Write each answer as a fraction and as decimal rounded to four places.



$$\sin S = \frac{63}{65} = .9692$$

$$\sin R = \frac{16}{65} = .2462$$

Example #2: Find cos U and cos W. Write each answer as a fraction and as a decimal round to four places.



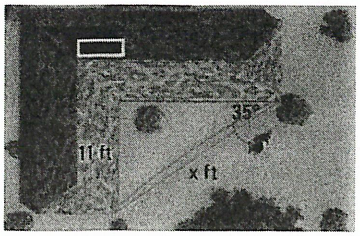
$$\cos U = \frac{18}{30} = \frac{3}{5} = .6$$

$$\cos W = \frac{24}{30} = \frac{4}{5} = .8$$

SOH - CAH - TOA

$\sin = \frac{\text{opp.}}{\text{hyp.}}$ $\cos = \frac{\text{adj}}{\text{hyp}}$ $\tan = \frac{\text{opp}}{\text{adj}}$

Example #3: You want to string cable to make a dog run from two corners of a building, as shown in the diagram. Write and solve a proportion using a trigonometric ratio to approximate the length of cable you will need



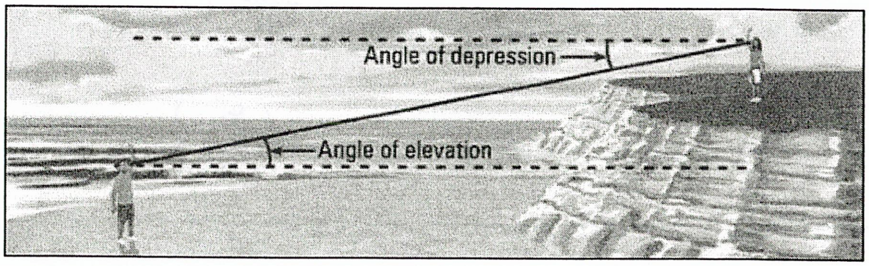
$$\frac{\sin 35}{1} = \frac{11}{x}$$

$$\frac{x \cancel{\sin 35}}{\cancel{\sin 35}} = \frac{11}{\sin 35}$$

$$x = \frac{11}{\sin 35}$$

$$\boxed{x \approx 19 \text{ ft}}$$

Definitions:

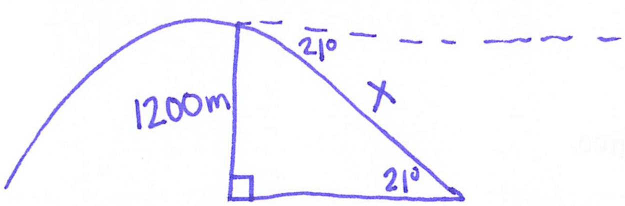


<u>Angle of Elevation:</u> When <u>you look up</u> at an object, the angle your line of sight makes with a horizontal line.

<u>Angle of Depression:</u> When <u>you look down</u> at an object, the angle your line of sight makes with a horizontal line

*****These angles are** <u>Congruent</u> *** (alt int ∠'s)

Example #4: you are skiing on a mountain with an altitude of 1200 meters. The angle of depression is 21°. About how far do you ski down the mountain?

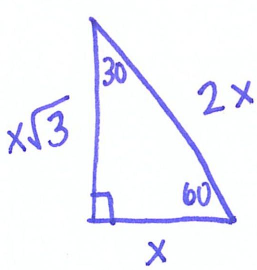


$$\frac{\sin 21}{1} = \frac{1200}{x}$$

$$\frac{x \sin 21}{\sin 21} = \frac{1200}{\sin 21}$$

$$\boxed{x \approx 3348 \text{ m}}$$

Example #5: Use a special right triangle to find the sine and cosine of a 60° angle.



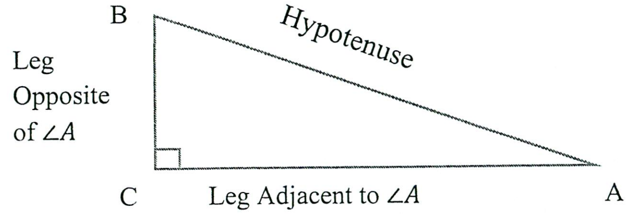
$$\sin 60° = \frac{x\sqrt{3}}{2x} = \frac{\sqrt{3}}{2}$$

$$\cos 60° = \frac{x}{2x} = \frac{1}{2}$$

Chapter 7.7: Solve Right Triangles

Inverse Trigonometric Ratios:

Let $\angle A$ be an acute angle.



Inverse Tangent: If $\tan A = x$, then $\tan^{-1} x = m\angle A$ $\rightarrow$ $\tan^{-1}\left(\frac{BC}{AC}\right) = m\angle A$

Inverse Sine: If $\sin A = y$, then $\sin^{-1} y = m\angle A$ $\rightarrow$ $\sin^{-1}\left(\frac{BC}{AB}\right) = m\angle A$

Inverse Cosine: If $\cos A = z$, then $\cos^{-1} z = m\angle A$ $\rightarrow$ $\cos^{-1}\left(\frac{AC}{AB}\right) = m\angle A$

Example #1: Find the approximate $m\angle A$ to the nearest tenth of a **degree** using…

a. Tangent $\tan^{-1}\left(\frac{16}{20}\right) = 38.7°$

$16^2 + 20^2 = x^2$

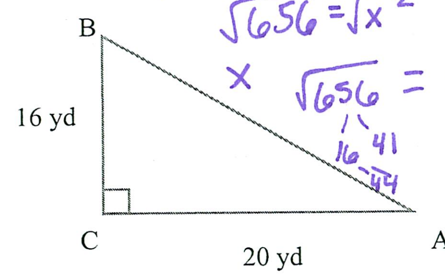
$256 + 400 = x^2$

$\sqrt{656} = \sqrt{x^2}$

$\sqrt{656} = x$

$4\sqrt{41} = x$

b. Sine $\sin^{-1}\left(\frac{16}{4\sqrt{41}}\right) = 38.7°$



c. Cosine $\cos^{-1}\left(\frac{20}{4\sqrt{41}}\right) = 38.7°$

d. Find the approximate $m\angle B$ to the nearest tenth of a degree.

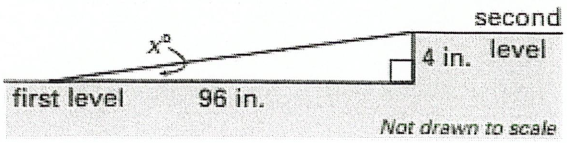
$m\angle B = 90 - 38.7°$

$m\angle B = 51.3°$

e. Find the length of AB.

$AB = 4\sqrt{41} \approx 25.6$

Example #2: You are building a track for a model train. You want the track to incline from the first level to the second level, 4 inches higher, in 96 inches. Is the angle of elevation less than 3°?

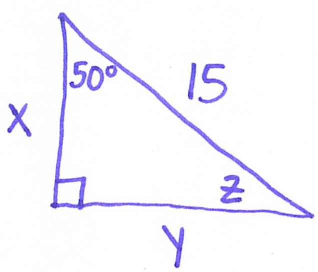


$\tan^{-1}\left(\frac{4}{96}\right) = 2.4°$

$2.4 < 3$

yes

Example #3: Solve a right triangle that has a 50° angle and a 15 inch hypotneuse.



$90 - 50 = z$

$\boxed{40° = z}$

$15(\cos(50)) = \left(\frac{x}{15}\right)15$

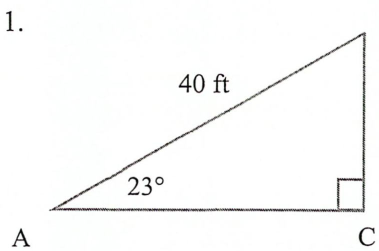
$\boxed{x = 9.6}$

$15(\sin 50) = \left(\frac{y}{15}\right)15$

$\boxed{y = 11.5}$

Concept Check: Solve the right triangles. Round decimal answers to the nearest tenth.

1.



$m\angle B = 90 - 23$

$\boxed{m\angle B = 67°}$

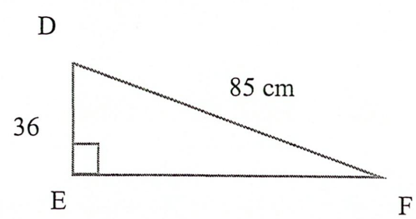
$40(\cos 23) = \left(\frac{AC}{40}\right)40$

$\boxed{36.8 = AC}$

$40(\sin 23) = \left(\frac{BC}{40}\right)40$

$\boxed{15.6 = BC}$

2.



$36^2 + EF^2 = 85^2$

$1296 + EF^2 = 7225$

$-1296 \qquad -1296$

$\sqrt{EF^2} = \sqrt{5929}$

$\boxed{EF = 77}$

$m\angle D = \cos^{-1}\left(\frac{36}{85}\right)$

$\boxed{m\angle D = 64.9°}$

$m\angle F = 90 - 64.9$

$\boxed{m\angle F = 25.1°}$