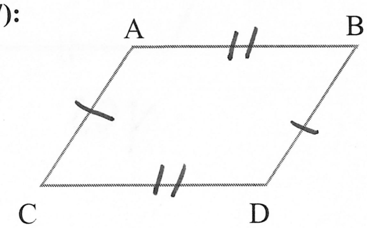
Chapter 8.3: Use Properties of Parallelograms

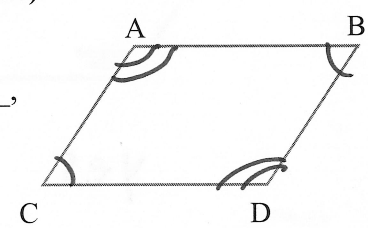
Parallelogram with Congruent Sides Converse Theorem (Theorem 8.7):
(Converse of Theorem 8.3)

If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.



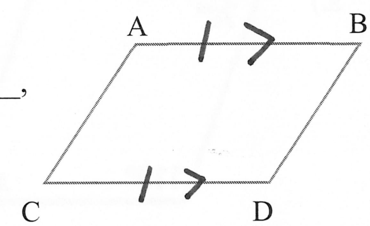
Parallelogram with Congruent Angles Converse Theorem (Theorem 8.8):
(Converse of Theorem 8.4)

If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.



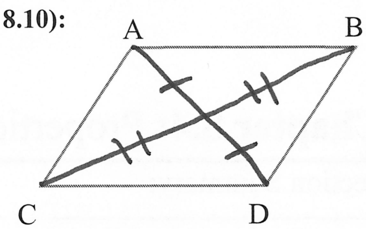
Quadrilateral with a Congruent and Parallel Side (Theorem 8.9):

If one pair of opposite sides of a quadrilateral are congruent and parallel, then the quadrilateral is a parallelogram.



Parallelogram with Bisecting Diagonal Converse Theorem (Theorem 8.10):
(Converse of Theorem 8.6)

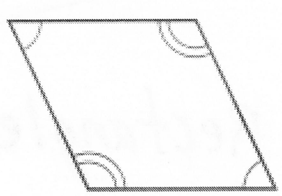
If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram



Example #1: Decide whether you are given enough information to determine that the quadrilateral is a parallelogram. Explain your reasoning.

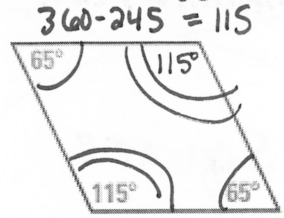
$115+65+65$
$360-245 = 115$

a.)



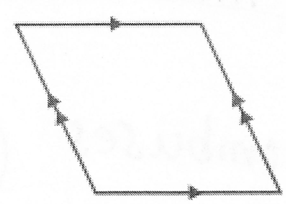
yes
thm 8.8

b.)



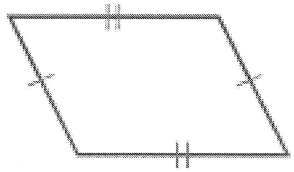
yes

c.)



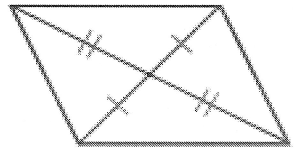
yes

d.)



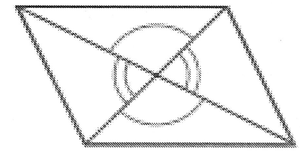
yes

e.)



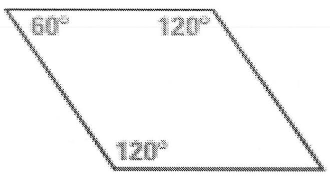
yes

f.)



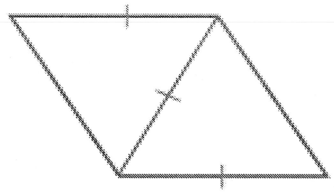
NO

g.)



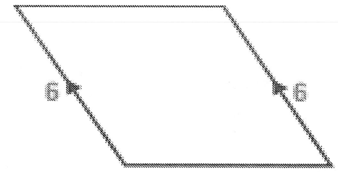
yes

h.)



NO

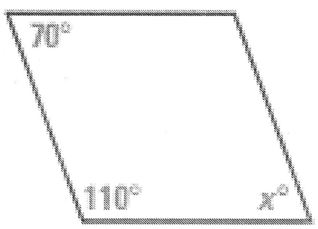
i.)



yes

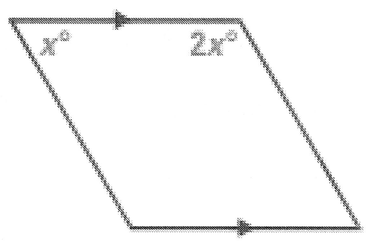
Example #2: What value of *x* will make the polygon a parallelogram?

a.)



70°

b.)

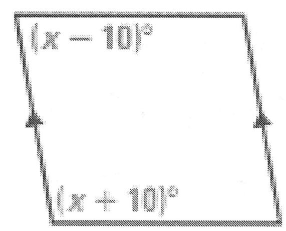


$x + 2x = 180$

$\frac{3x}{3} = \frac{180}{3}$

$\boxed{x = 60}$

c.)



$x - 10 + x + 10 = 180$

$\frac{2x}{2} = \frac{180}{2}$

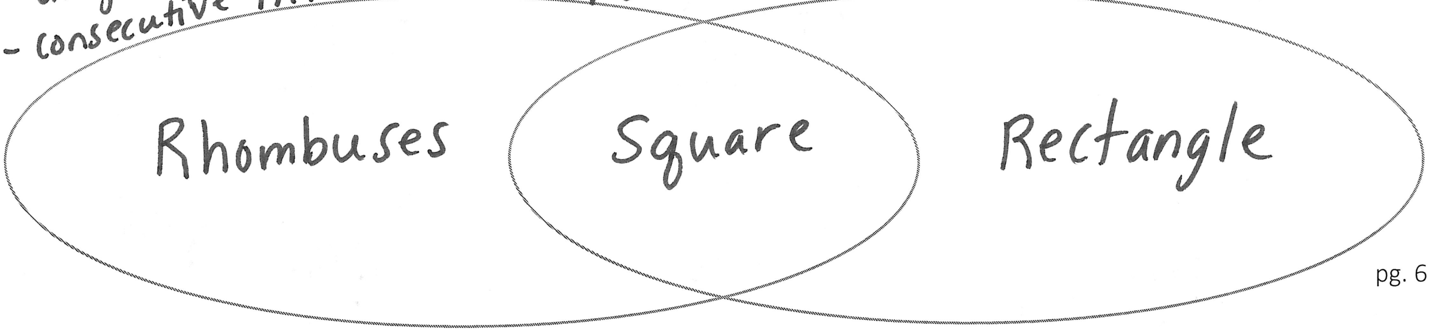
$\boxed{x = 90°}$

Chapter 8.4: Properties of Rhombuses, Rectangles and Squares

Section Summary:

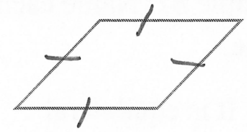
Parallelograms

- All int ∠'s add up to 360°
- diagonals bisect each other
- consecutive int ∠'s are supplementary (180)
- 2 pairs of parallel sides
- opp. ∠'s are ≅
- opp sides are ≅



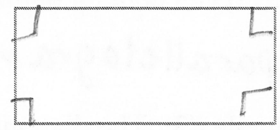
Rhombus Corollary:

A quadrilateral is a rhombus iff it has four congruent sides



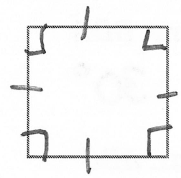
Rectangle Corollary:

A quadrilateral is a rectangle iff it has four congruent angles



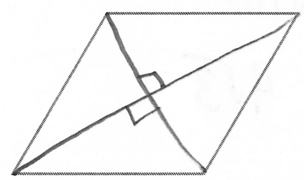
Square Corollary:

A quadrilateral is a square iff it is a rhombus and a rectangle



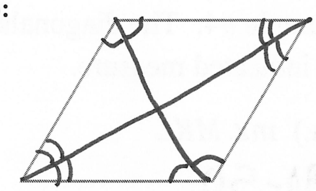
Parallelogram with Perpendicular Diagonals Theorem (Theorem 8.11):

A parallelogram is a rhombus iff its diagonals are perpendicular



Parallelogram with Diagonals Bisecting Angles Theorem (Theorem 8.12):

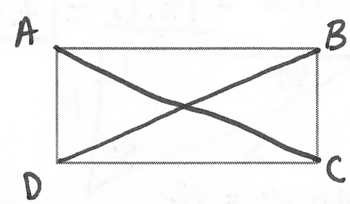
A parallelogram is a rhombus iff each diagonal bisects a pair of opposite angles



Parallelogram with Congruent Diagonals Theorem (Theorem 8.13):

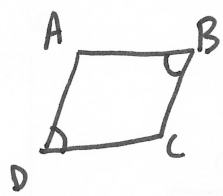
A parallelogram is a rectangle iff its diagonals are congruent

$AC = BD$



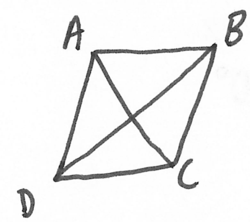
Example #1: For any rhombus ABCD, decide whether the statement is *always* or *sometimes* true. Draw a diagram and explain your reasoning.

a.) $\angle ABC \cong \angle CDA$



Always
rhombus is
a parallelogram

b.) $\overline{CA} \cong \overline{DB}$



Sometimes
if the shape
is a square

Example #2: Name each quadrilateral- *parallelogram, rectangle, rhombus* and *square*- for which the statement is true.

a.) It is equilateral

rhombus, square

b.) The diagonals are congruent

rectangle, square

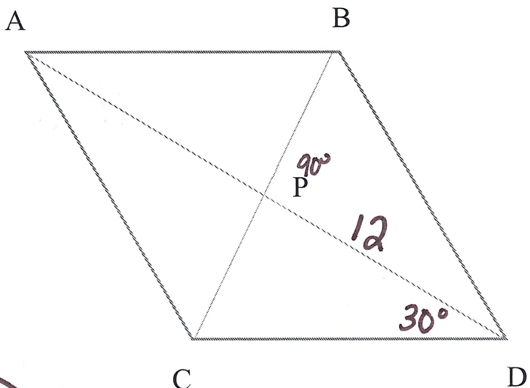
c.) It can contain obtuse angles

parallelogram, rhombus

d.) It contains no acute angles

rectangle, square

Example #3: The diagonals of rhombus ABCD interest at P. Given that $m\angle ADC = 30°$ and $DP = 12$, find the indicated measure.



a.) $m\angle BDA$

30°

b.) $m\angle BPD$

90°

c.) $m\angle ABC$

60°

d.) DA

24

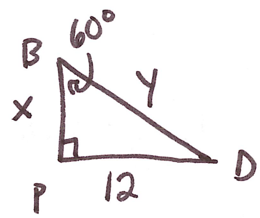
e.) BP

$4\sqrt{3}$

f.) BD

$4\sqrt{3}\cdot 2$

$8\sqrt{3}$



hyp = 2x

$y = 2\cdot 4\sqrt{3}$

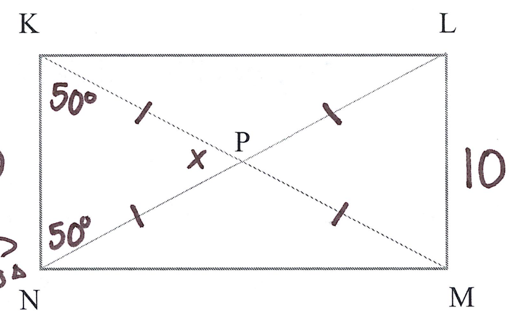
30 - 60 - 90

$\frac{x\sqrt{3}}{\sqrt{3}} = \frac{12}{\sqrt{3}}$

$x = \frac{12}{\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right)$

$x = \frac{12\sqrt{3}}{3} = 4\sqrt{3}$

Example #4: The diagonals of rectangle KLMN interest and P. Given that $m\angle MKN = 50°$ and $LM = 10$, find the indicated measure.



because it's → isosceles△

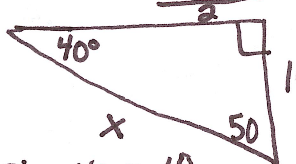
a.) $m\angle MKL$

90-50

b.) $m\angle KPN$

$180 - 50 - 50 = \boxed{80°}$

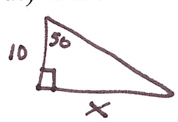
c.) PM $= \frac{15.6}{2} = \boxed{7.8}$



$\frac{\sin 40}{1} = \frac{10}{x}$

$\frac{x \sin 40}{\sin 40} = \frac{10}{\sin 40}$

d.) MN

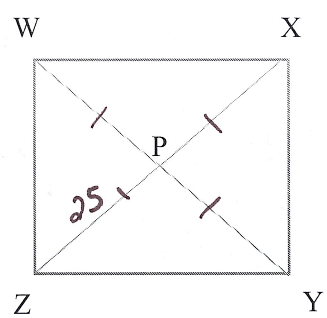


$10\,\frac{(\tan 50)}{1} = \frac{x}{10}\cdot 10$

$10 \tan 50 = x$

$\boxed{x = 11.9}$

Example #5: The diagonals of square WXYZ interest and P. Given that $PZ = 25$, find the indicated measure.



a.) $m\angle WPZ$

90°

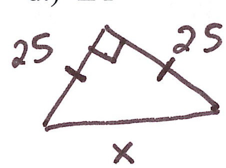
b.) $m\angle WXP$

45°

c.) PY

25

d.) ZY



45 - 45 - 90

$\boxed{x = 25\sqrt{2}}$