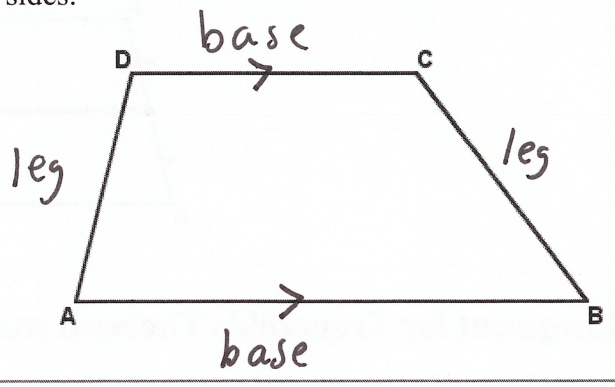
Chapter 8.5: Use Properties of Trapezoids and Kites

Trapezoid: a quadrilateral with exactly one pair of parallel sides.

Bases: parallel sides

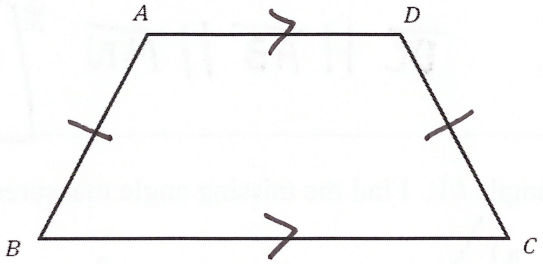
Base Angles: 2 pairs, $\angle A, \angle B$ $\angle D, \angle C$

Legs: non-parallel sides



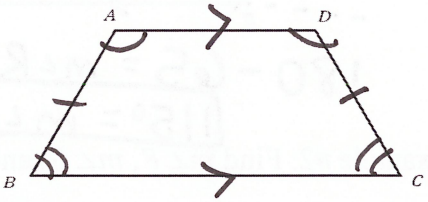
Isosceles Trapezoid:

A trapezoid with congruent legs



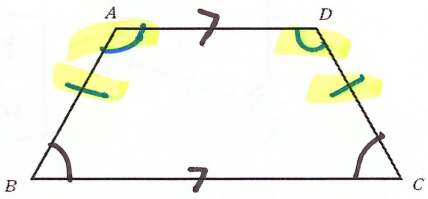
Isosceles Trapezoid Theorem (Theorem 8.14):

If a trapezoid is isosceles, then each pair of base angels are congruent



Trapezoid with Congruent Base Angles Theorem (Theorem 8.15):

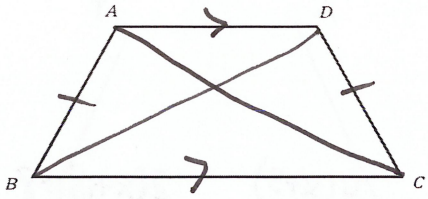
If a trapezoid has a pair of congruent base angles, then it is an isosceles trapezoid



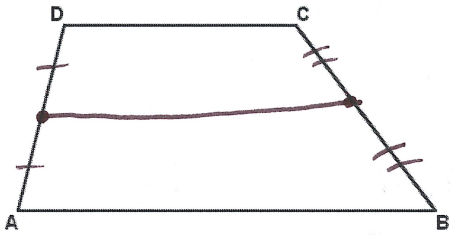
Trapezoid with Congruent Diagonals Theorem (Theorem 8.16):

A trapezoid is isosceles iff its diagonals are congruent.

$\overline{AC} \cong \overline{BD}$



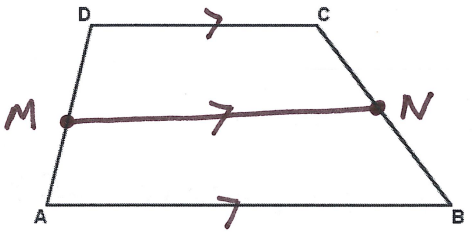
Midsegment of a Trapezoid: The segment that connects the midpoints of its legs.



Midsegment for Trapezoids Theorem (Theorem 8.17):

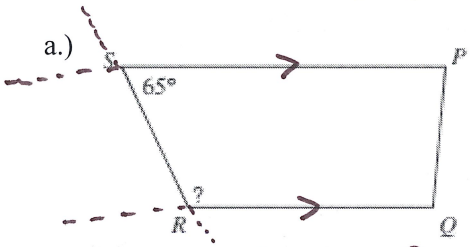
Midsegment of a trapezoid is parallel to each base and its length is half the sum of the 2 bases.

$\overline{DC} \parallel \overline{AB} \parallel \overline{MN}$ * $MN = \frac{DC + AB}{2}$



Example #1: Find the missing angle measures in the trapezoids below.

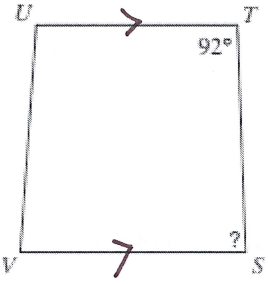
a.)



$180 - 65 = m\angle R$

$115° = m\angle R$

b.)

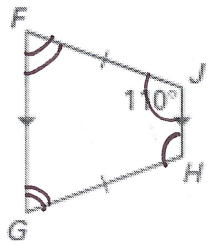


$180 - 92 = m\angle S$

$88° = m\angle S$

Example #2: Find $m\angle F$, $m\angle G$, and $m\angle H$

a.)



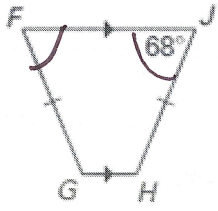
$m\angle H = 110°$

$180 - 110 = 70°$

$m\angle F = 70°$

$m\angle G = 70°$

b.)



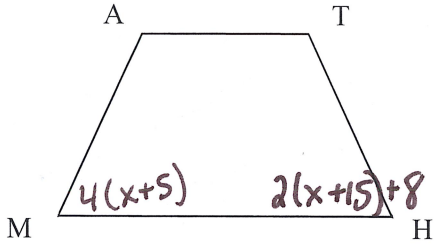
$m\angle F = 68°$

$180 - 68 = m\angle H$

$112° = m\angle H$

$112° = m\angle G$

Example #3: In isosceles trapezoid MATH, the $m\angle M = 4(x + 5)°$ and $m\angle H = 2(x + 15) + 8°$. Find the measure of all 4 angles of the trapezoid.



$4(x+5) = 2(x+15) + 8$

$4x + 20 = 2x + 30 + 8$

$-2x \quad -20 \quad -2x \quad -20$ (38 − 20)

$\frac{2x}{2} = \frac{18}{2}$

$x = 9$

$m\angle M = 4(9+5)$

$= 4(14)$

$m\angle M = 56° \quad m\angle H = 56°$

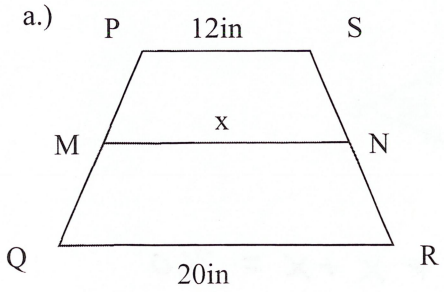
$180 - 56 = m\angle A$

$124° = m\angle A$

$124° = m\angle T$

Example #4: In the diagrams below, $\overline{MN}$ is the midsegment of trapezoid PQRS. Solve for x.

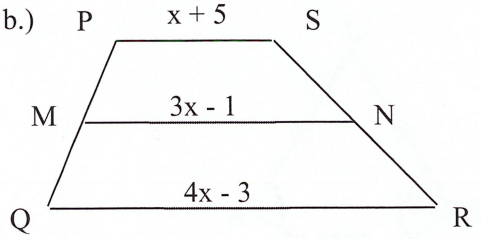
a.)



$MN = \frac{12 + 20}{2}$

$MN = 16$ in

b.)



$\frac{x+5+4x-3}{2} = 3x-1$

$2\left(\frac{5x+2}{2}\right) = (3x-1)2$

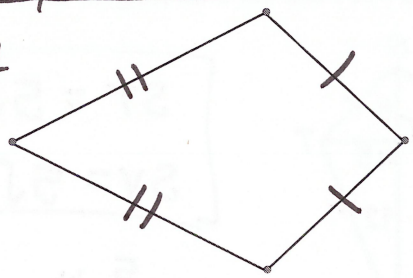
$5x + 2 = 6x - 2$

$-5x \quad -5x$

$2 = x$

Kite: A quadrilateral that has…

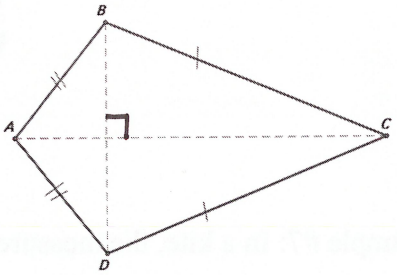
- 2 pairs of consecutive sides congruent
- But opposite sides are not congruent



Quadrilateral with Perpendicular Diagonals Theorem (Theorem 8.18):

If a quadrilateral is a kite, then its diagonals are perpendicular

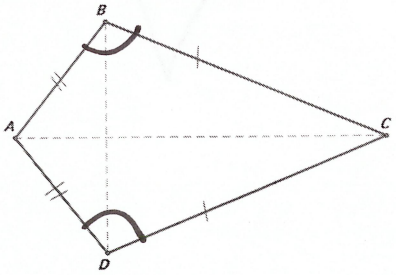
$\overline{AC} \perp \overline{BD}$



Quadrilateral with One Pair of Opposite Congruent Angles Theorem (Theorem 8.19):

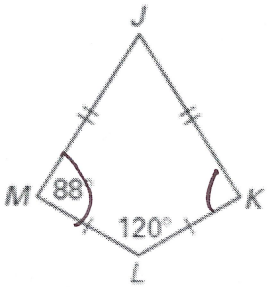
If a quadrilateral is a kite, then exactly 1 pair of opposite angles are congruent.

$m\angle B \cong m\angle D$ and $m\angle A \ncong m\angle C$



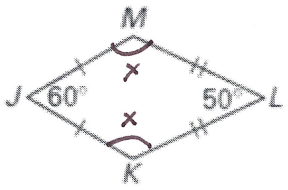
Example #5: JKLM is a kite. Find the $m\angle K$

a.)



$m\angle K = 88°$

b.)



$60 + 50 + x + x = 360$

$110 + 2x = 360$

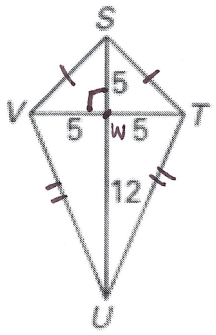
$-110 \quad -110$

$\frac{2x}{2} = \frac{250}{2}$

$x = 125$

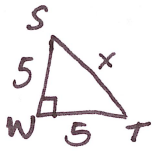
$m\angle K = 125°$

Example #6: Find the side lengths of Kite STUV



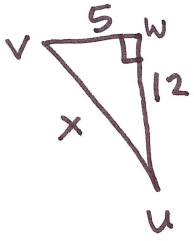
$ST = 5\sqrt{2}$ $\quad TU = 13$

$SV = 5\sqrt{2}$ $\quad VU = 13$



45-45-90

$x = 5\sqrt{2}$



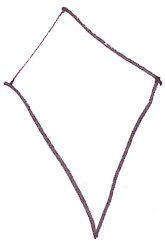
$5^2 + 12^2 = x^2$

$25 + 144 = x^2$

$\sqrt{169} = \sqrt{x^2}$

$13 = x$

Example #7: In a kite, the measures of the angles are $3x°$, 75°, 90° and 120°. Find the value of x.



$3x + 75 + 90 + 120 = 360$

$3x + 285 = 360$

$-285 \quad -285$

$\frac{3x}{3} = \frac{75}{3}$

$x = 25$

Chapter 8.6: Identify Special Quadrilaterals

Quadrilateral Hierarchy

Quadrilaterals
- 4 sides / angles
- all int ∠'s add up to 360°

trapezoids
- bases are parallel (one set)

isosceles trapezoid

- legs are ≅
- diagonals are ≅
- base angles pairs are ≅
- midsegment = average of the bases

$$\text{midseg} = \frac{b_1 + b_2}{2}$$

parallelograms
- 2 pairs of // opp. sides
- 2 pairs of ≅ opp angles
- 2pairs of ≅ opp. sides
- consecutive ∠'s are supple.
- diagonals bisect

Kite
- diagonals are ⊥
- one pair of opp. angles ≅
- 2 pair of ≅ consecutive sides
- opp. sides ≇

rhombus
- 4 ≅ sides
- diagonals are ⊥ (perpendicular)
- diagonals bisect ∠'s

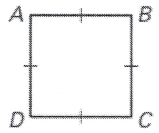
rectangles
- diagonals are ≅
- all ∠'s = 90°

square

Example #1: Quadrilateral JKLM has both pairs of opposite angles congruent. What types of quadrilateral meet this condition?

parallelograms, rectangle, rhombus, squares

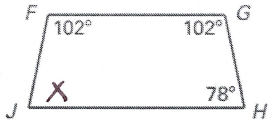
Example #2: What is the most specific name for quadrilateral ABCD?



rhombus - 4 ≅ sides

no proof of ∠'s = 90

Example #3: Is there enough information given in the diagram to show that quadrilateral FGHJ is an isosceles trapezoid? Explain



$\angle F \cong \angle G$ one pair of base ∠'s congruent

$102 + 78 = 180$ ∠G and ∠H are supplementary

$\overline{FG} \parallel \overline{JH}$

$360 = 102 + 102 + 78 + x$

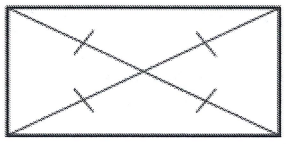
$-282 \quad -282$

$78° = x$

$\angle J \cong \angle H$ yes

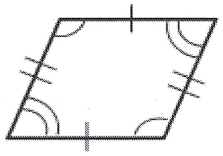
Example #4: What is the most specific name for each quadrilateral?

a.)



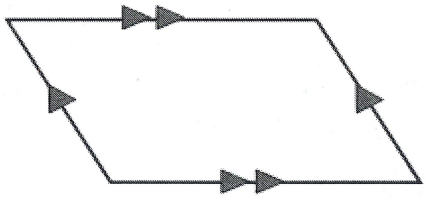
rectangle

b.)



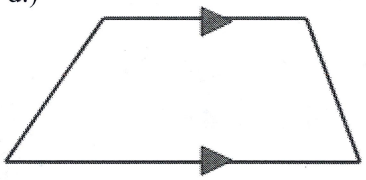
parallelogram

c.)



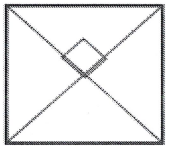
parallelogram

d.)



trapezoid

e.)

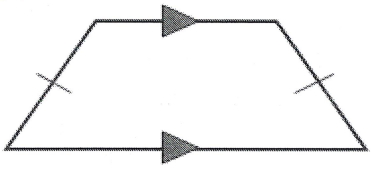


quadrilateral

f.)

quadrilateral

g.)



isosceles trapezoid