Geometry

Ms. Linzmeier

Unit 10: Properties of Circles

Priority Standard: G-C 1-5: Understand and apply theorems about circles. Find arc lengths and areas of sectors of circles.

Unit 10 "I can" Statements:

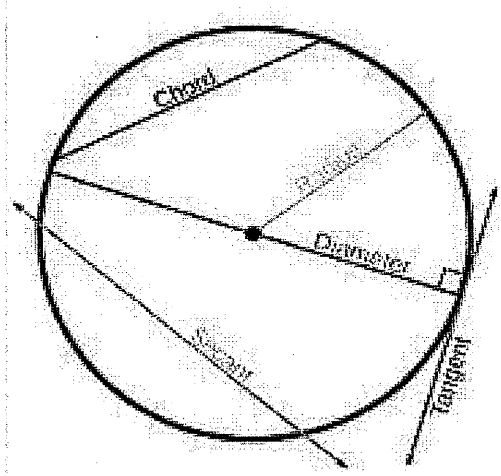
1. I can use properties of a tangent to a circle.
2. I can use angle measures to find arc measures.
3. I can use relationships of arcs and chords in a circle.
4. I can use inscribed angles of circles
5. I can find the measures of angles inside or outside a circle.
6. I can find segment lengths in circles.
7. I can write equations of circles in the coordinate plane.

Chapter 10.1: Use Properties of Tangents

Goal: To be able to identify chords, tangents, secants, radii and diameters of circles and use properties of a tangent to a circle.

A circle is the set of all points in a plane that are ______________________

from a given point called the ________________ of the circle.



A segment whose endpoints are the center and any point on the circle is a ____________________ .

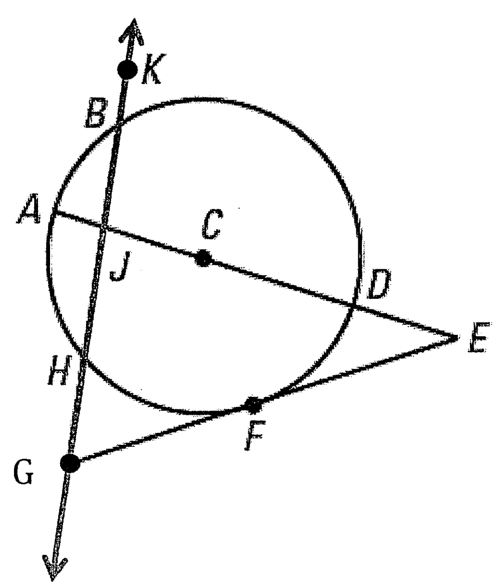
A _________________ is a segment whose endpoints are on a circle.

A _________________ is a chord that contains the center of the circle.

A _________________ is a line that intersects a circle in two points.

A _________________ is a line in the plane of a circle that intersect the circle in exactly one point, called the ________________________________ .

Example #1: Tell whether the line or segment is best described as a chord, a secant, a tangent, a diameter, or a radius of $\odot C$.



a. $\overline{AD}$

b. $\overline{CD}$

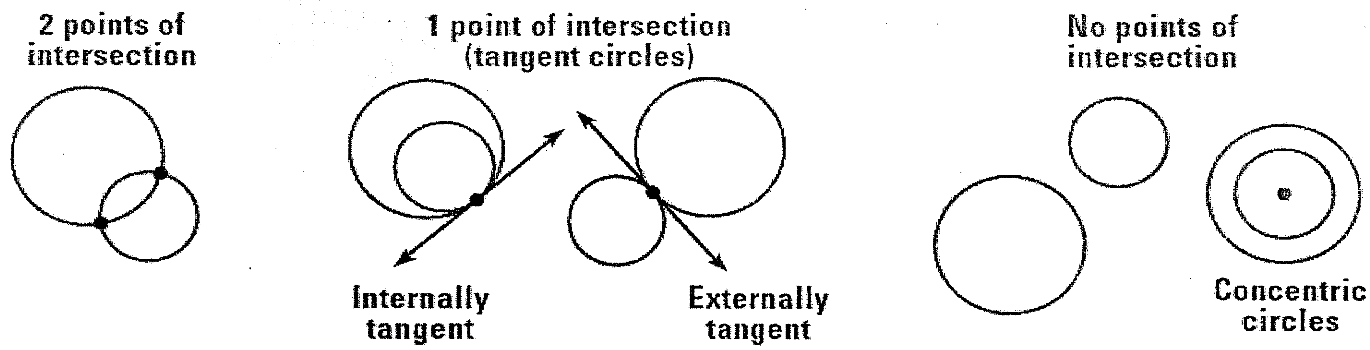
c. $\overleftrightarrow{EG}$

d. $\overline{HB}$

e. $\overleftrightarrow{GK}$

In a plane, two circles can intersect in ____________________ points, ______________________ point or

_____________________ points.



Coplanar circles that intersect in one point are called _________________________________.

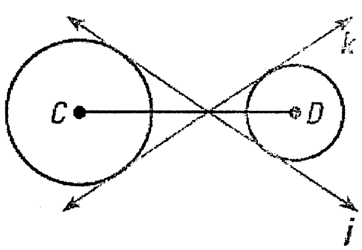
Coplanar circles that have a common center are called _________________________________.

A line, ray or segment that is tangent to two coplanar circles is called ________________________________.

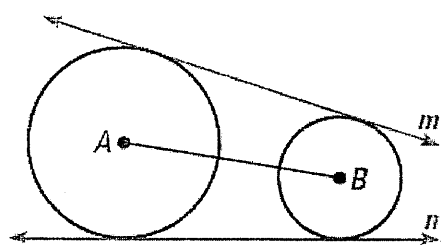
- A _______________________________ intersects the segment that joins the centers of the two circles.
- A _______________________________ does not intersect the segment that joins the centers of the two circles.

Example #2: Tell whether the common tangents are internal or external.

a.

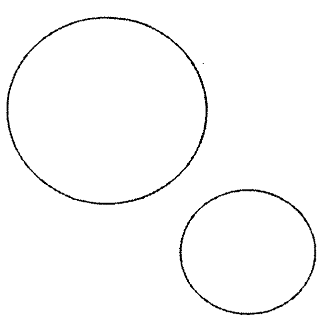


b.

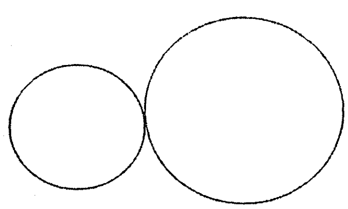


Example #3: Tell how many common tangents the circles have and draw them.

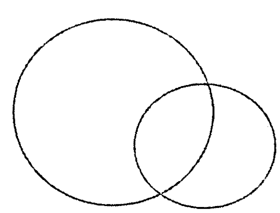
a.



b.

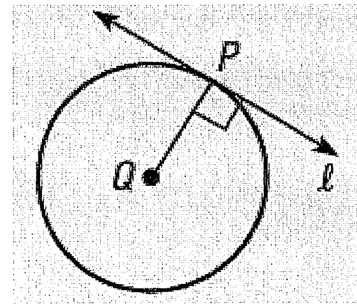


c.

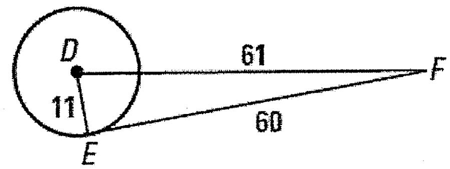


Theorem 10.1

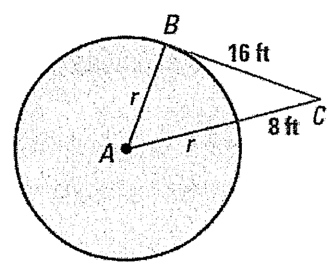
In a plane, a line is tangent to a circle if and only if the line is perpendicular to a radius of the circle at its endpoint on the circle.



Example #4: Verify that $\overleftrightarrow{EF}$ is tangent to ⊙ D

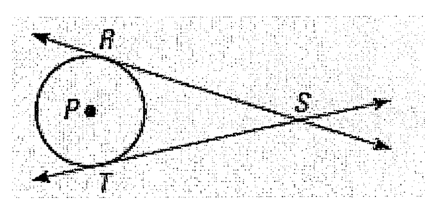


Example #5: You are standing at C, 8 feet from a grain silo. The distance from you to a point of tangency on the tank is 16 feet. What is the radius of the silo?

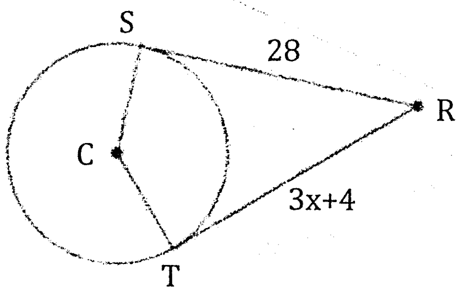


Theorem 10.2

Tangent segments from a common external point are congruent.



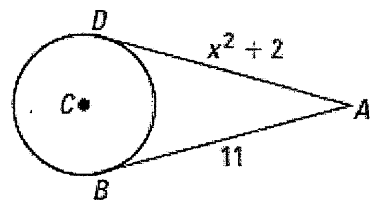
Example #6: $\overline{RS}$ is tangent to $\odot C$ at S and $\overline{RS}$ is tangent to $\odot C$ at T. Find the value of x



Example #7:

$\overleftrightarrow{AB}$ is tangent to $\odot C$ at B.
$\overleftrightarrow{AD}$ is tangent to $\odot C$ at D.

Find the value of x.

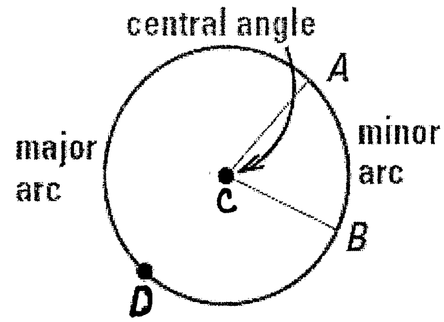


Chapter 10.2: Find Arc Measures

Goal: Be able to use angle measures to find arc measures

Arc Measures:

A **central angle** of a circle is an angle whose _______________ is the center of the circle.



If $m\angle ACB$ is less than ___________, then the points on $\odot C$ that lie in the interior of $\angle ACB$ form a _____________________________ with endpoints A and B. (The measure of a minor arc is the measure of its central angle.)

Naming:

The points on $\odot C$ that do not lie on minor arc $\overset{\frown}{AB}$ form a _________________________with endpoints A and B. (The measure of the entire circle is 360°. The measure of a major arc is the difference between 360° and the measure of the related minor arc)

Naming:

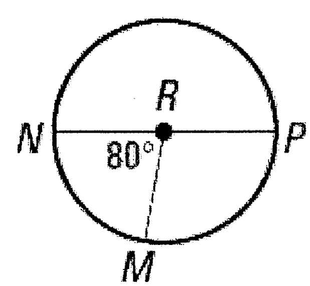
A ______________________________ is an arc with endpoints that are the endpoints of a diameter.

Example #1: For each arc of ⊙.: Identify as a minor arc, major arc or semicircle and find its measure.

a. $\overset{\frown}{MN}$

b. $\overset{\frown}{MPN}$

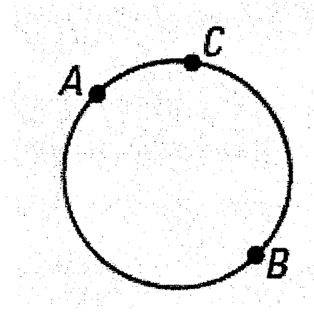
c. $\overset{\frown}{PMN}$



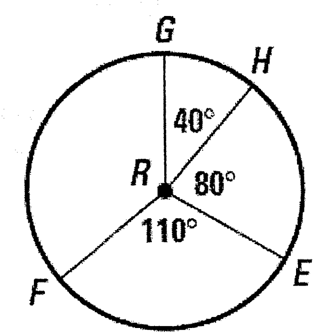
Two arcs of the <u>same circle</u> are **adjacent** if they intersect at exactly <u>one point</u>. You can add the measures of adjacent arcs.

Arc Addition Postulate (Postulate 26):

The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs.



Example #2: Find the measure of each arc.



a. $\overset{\frown}{GE}$

b. $\overset{\frown}{GEF}$

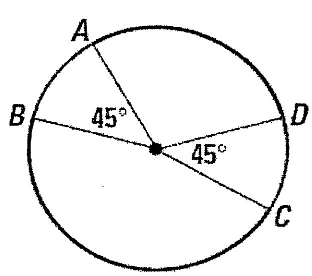
c. $\overset{\frown}{GF}$

d. $\overset{\frown}{EFG}$

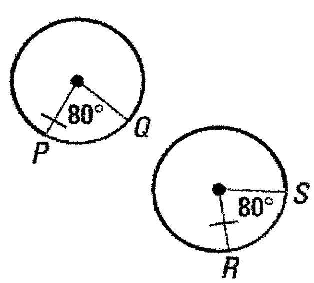
Two circles are **congruent circles** if they have the same _______________. Two arcs of the same circle or of congruent circles are **congruent arcs** if they have the _______________ measure. So, two minor arcs of the same circle or of congruent circles are congruent if their central angles are ________________________.

Example #3: Are the given arcs congruent?

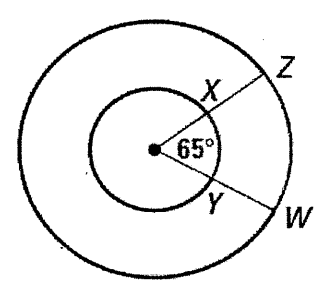
a.



b.

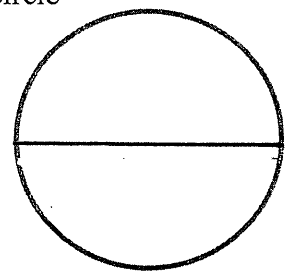


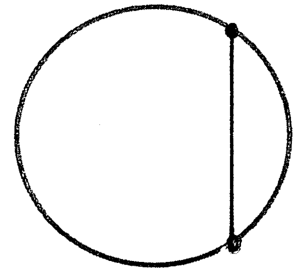
c.



Chapter 10.3: Apply Properties of Chords

Goal: I will be able to use relationships of arcs and chords in a circle



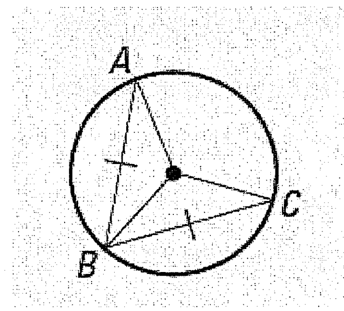


Recall: A chord is a segment with endpoints on a circle. Because its endpoints lie on the circle, any chord divides the circle into two arcs. A **diameter** divides the circle into two ___________________________.

Any other chord divides a circle into a **minor arc** and a **major arc.**

Theorem 10.3:

In the same circle, or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent.

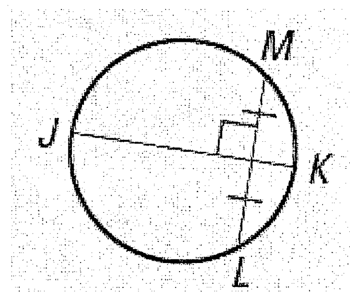


Theorem 10.4:

If one chord is a perpendicular bisector of another chord, then the first chord is a diameter.

If __________ is a perpendicular bisector of __________,

then __________ is a diameter of the circle.

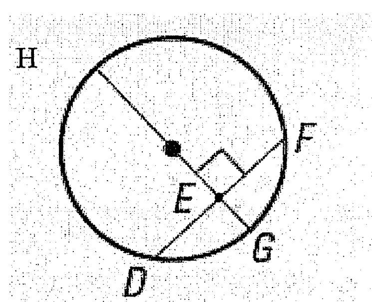


Theorem 10.5:

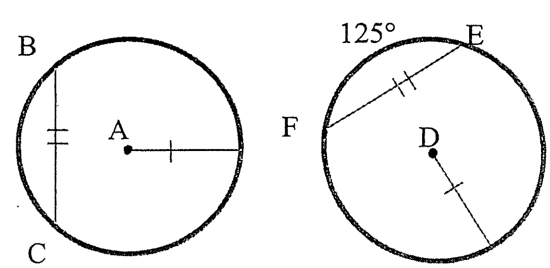
If a diameter of a circle is perpendicular to a chord, then the diameter bisects the chord and its arc.

If __________ is a diameter and ______________________,

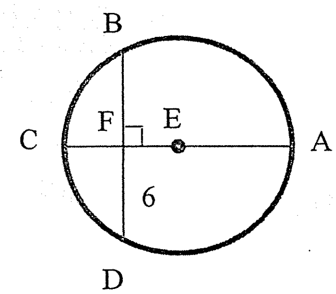
then ______________________ and ______________________,



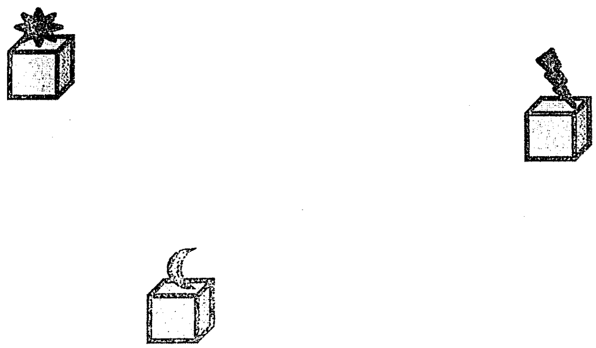
Example #1: In the diagram, $\odot A \cong \odot D$, $\overline{BC} \cong \overline{EF}$, and $m\widehat{EF} = 125°$. Find $m\widehat{BC}$.



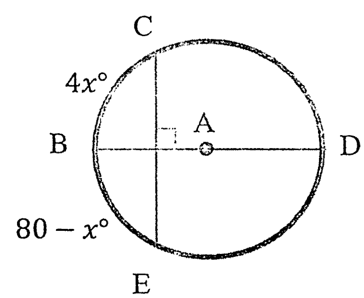
Example #2: Use the diagram of $\odot E$ to find the length of $\overline{BD}$ ..
Tell what theorem you use



Example #3: A journalist is writing a story about three sculptures, arranged as shown. Where should the journalist place a camera so that it is the same distance from each sculpture?

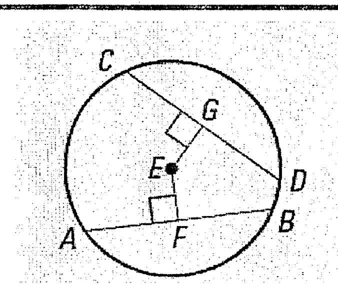


Example #4: Find the measures of $\widehat{CB}$, $\widehat{BE}$, and $\widehat{CE}$

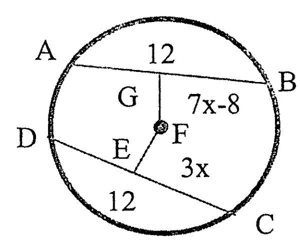


Theorem 10.6:

In the same circle, or in congruent circles, two chords are congruent if and only if they are equidistant from center.



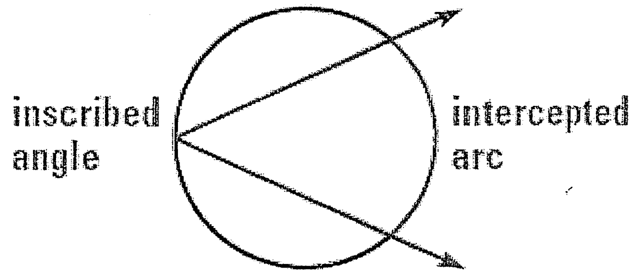
Example #5: In the diagram of $\odot F$, $AB = CD = 12$. Find EF.



Chapter 10.4: Use Inscribed Angles and Polygons

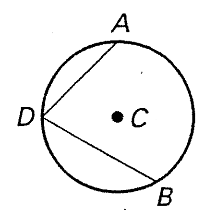
Goal: To be able to use inscribed angles of circles.

An **inscribed angle** is an angle whose vertex is on a circle and whose sides contain chords of the circle. The arc that lies in the interior of an inscribed angle and has endpoints on the angles is the ______________________________ of the angle.



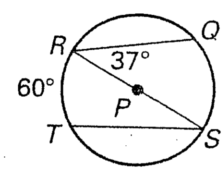
Measure of an Inscribed Angle Theorem (Theorem 10.7)

The measure of an inscribed angle is one half the measure of its intercepted arc.



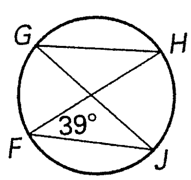
$m\angle ADB =$ ______________

Example #1: Find the indicated measure in ⊙ P.



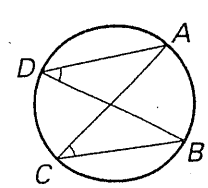
a. $m\angle S$ $\quad\quad$ $m\widehat{RQ}$

Example #2: Find $m\widehat{HJ}$ and $m\angle HGJ$. What do you notice about $\angle HGJ$ and $\angle HFJ$.



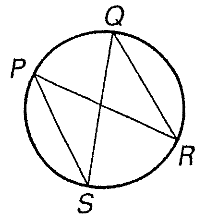
Theorem 10.8:

If two inscribed angles of a circle intercept the same arc, then the angles are congruent.



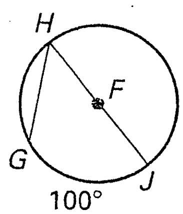
$\angle ADB \cong$ ______________

Example #3: Name two pairs of congruent angles in the figure.

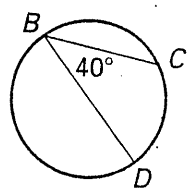


Checkpoint: Find the indicated measure.

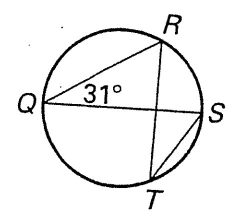
1. $m\angle GHJ$



2. $m\widehat{CD}$



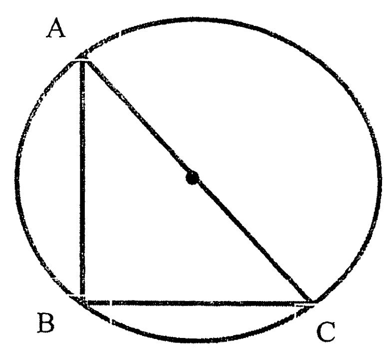
3. $m\angle RTS$



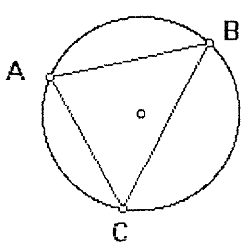
Theorem 10.9:

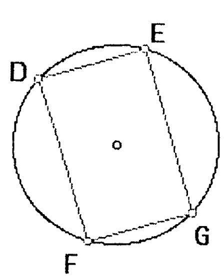
If a right triangle is inscribed in a circle, then the hypotenuse is a ____________________ of the circle. Conversely, if one side of an inscribed triangle is a diameter of the circle, then the triangle is a right triangle and the angle opposite the diameter is the right angle.

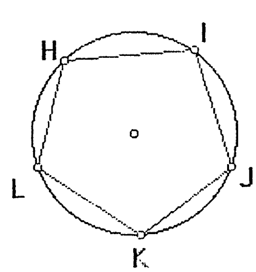
$m\angle ABC =$ ________ iff ________ is a diameter of $\odot D$.

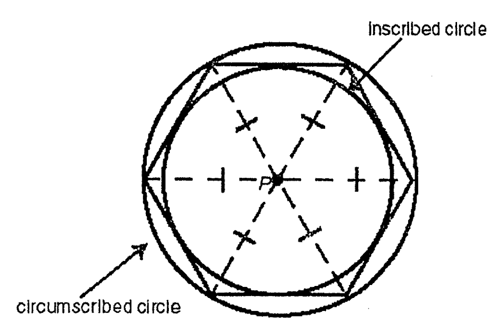


A polygon is an **inscribed polygon** if all of its ______________________ lie on a circle. The circle that contains the vertices a **circumscribed circle.**



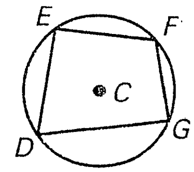






Theorem 10.10:

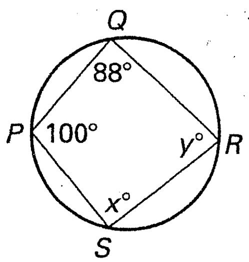
A quadrilateral can be inscribed in a circle iff its opposite angles are ______________________________.



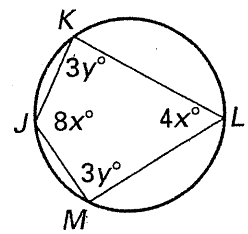
D, E, F and G lie on $\odot C$ iff $m\angle D + m\angle F =$ ______________________________

Example #4: Find the value of each variable.

a.

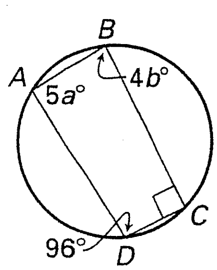


b.



Example #5: A right triangle is inscribed in a circle. The radius of the circle is 5.6 cm. What is the length of the hypotenuse of the right triangle?

Checkpoint: Find the values of *a* and *b*.

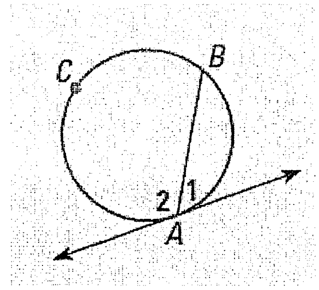


Chapter 10.5: Apply Other Angle Relationships in Circles

Goal: Be able to find the measures of angles inside or outside a circle.

Theorem 10.11:

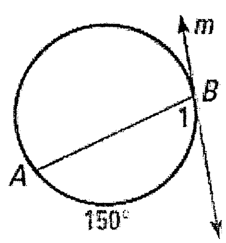
If a tangent and a chord intersect at a point on a circle, then the measure of each angle formed is ____________ the measure of its intercepted arc.



$m\angle 1 =$ __________ $m\angle 2 =$__________

Example #1: Line *m* is tangent to the circle. Find the measure of the red angle or arc.

a.



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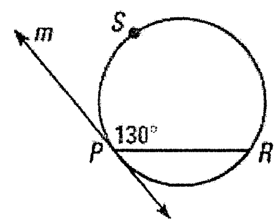
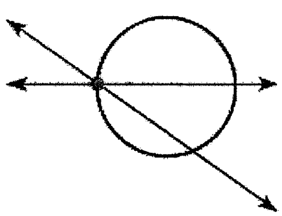
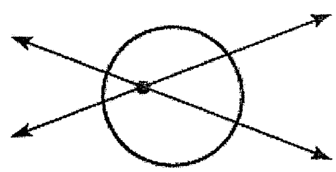


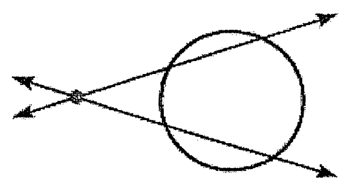
Intersecting Lines and Circles: If two lines intersect a circle, there are three places where the lines can intersect.



on the circle



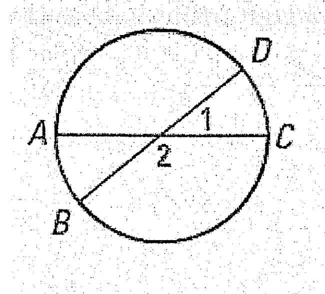
inside the circle



outside the circle

Angles <u>Inside</u> the Circle Theorem (Theorem 10.12):

If two chords intersect inside a circle, then the measure of each angle is one half the sum of the measures of the arcs intercepted by the angle and its vertical angle.

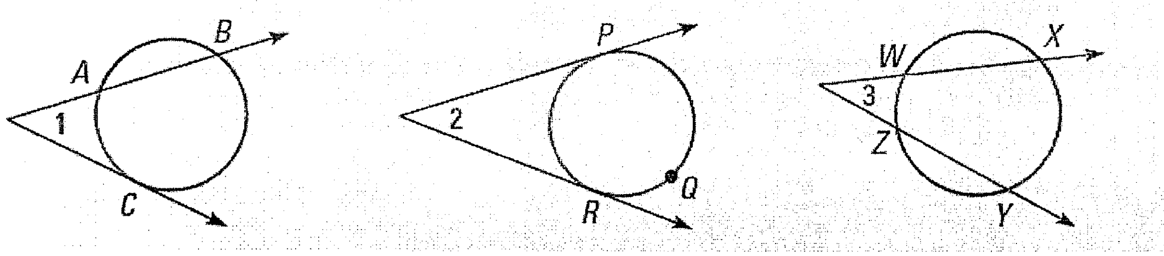


$m\angle 1 =$ ______________________

$m\angle 2 =$______________________

Angles Outside the Circle Theorem (Theorem 10.13):

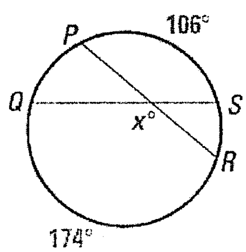
If a tangent and a secant, two tangents, or two secants intersect outside a circle, then the measure of the angle formed is one half the difference of the measures of the intercepted arcs.



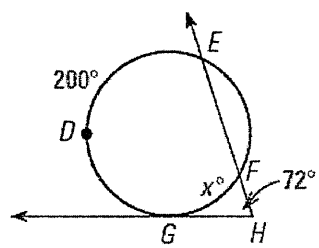
$m\angle 1 =$ ____________________ $m\angle 2 =$ ____________________ $m\angle 3 =$ ____________________

Example #2: Find the value of x.

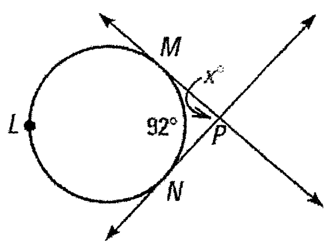
a.



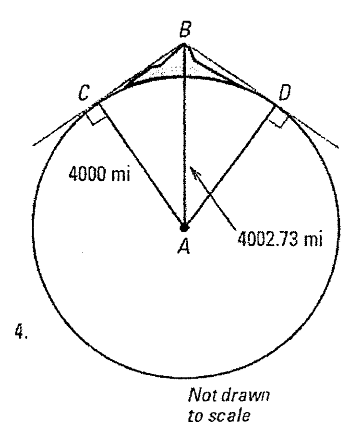
b.



c.

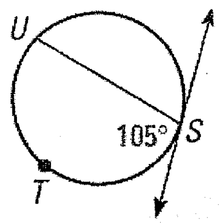


Example #3: You are on top of Mount Rainier on a clear day. You are about 2.73 miles above sea level. Find the measure of the arc $\overset{\frown}{CD}$ that represents the part of Earth that you can see.

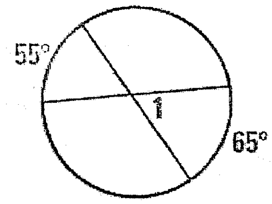


Checkpoint:

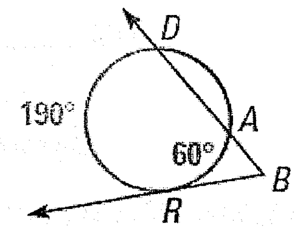
1. $m\widehat{STU}$



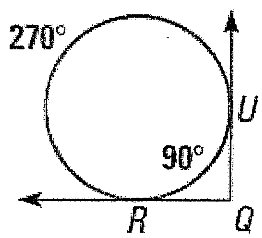
2. $m\angle 1$



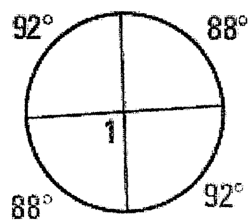
3. $m\angle DBR$



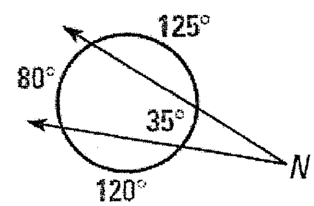
4. $m\angle RQU$



5. $m\angle 1$



6. $m\angle N$



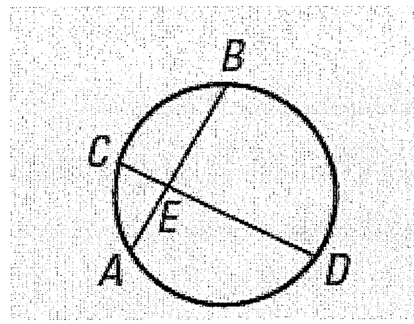
Chapter 10.6: Find Segment Lengths in Circles

** When two chords intersect in the interior of a circle, each chord is divided into two segments that are **

called ______________________________

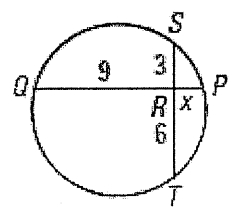
Segments of Chords Theorem (Theorem 10.14):

If two chords intersect in the interior of a circle, then the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord.

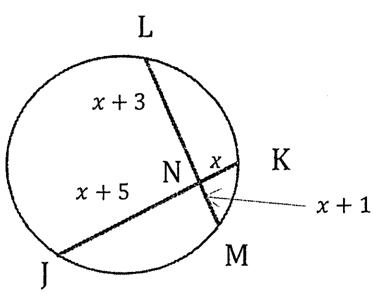


EA · EB = ______________

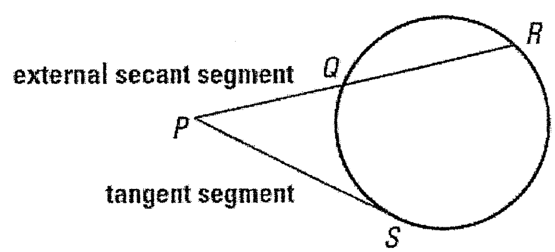
Example #1: Chords $\overline{ST}$ and $\overline{PQ}$ intersect inside the circle. Find the value of x.



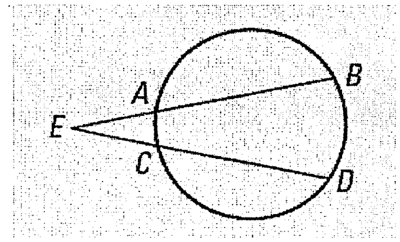
Example #2: Find ML and JK



A ______________________________ is a segment that contains a chord of a circle, and has exactly one endpoint outside the circle. The part of the secant segment that is outside the circle is called an ______________________________.



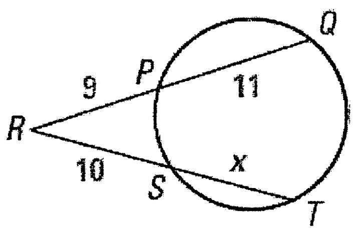
Segments of Secants Theorem **(Theorem 10.15):**



If two secant segments share the same endpoint outside a circle, then the product of the lengths of one secant segment and its external segment equals the product of the lengths of the other secant segment and its external segment.

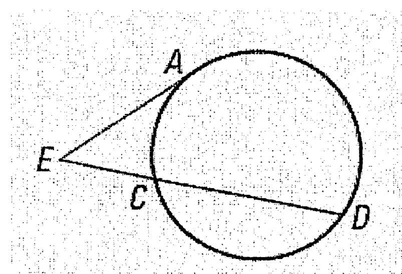
EA · EB = ________________

Example #3: Find the value of *x*.



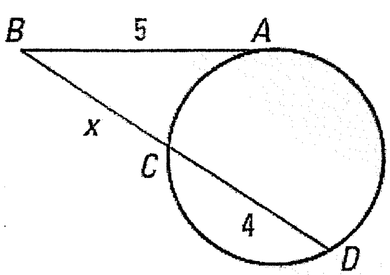
Segments of Secants and Tangents Theorem **(Theorem 10.16):**

If a secant segment and a tangent segment share an endpoint outside a circle, then the product of the lengths of the secant segment and its external segment equals the square of the lengths of the tangent segment.



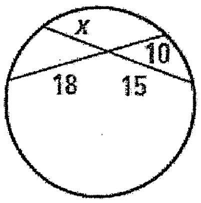
$(EA)^2$ = ________________

Example #4: Find the value of *x*.

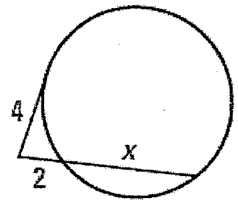


Checkpoint: Find the value of x.

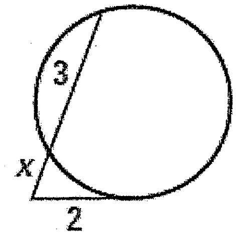
1.



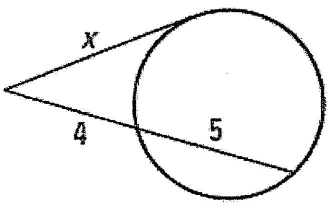
2.



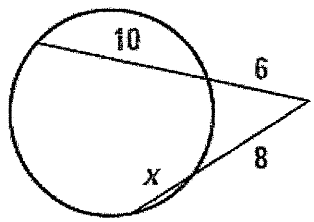
3.



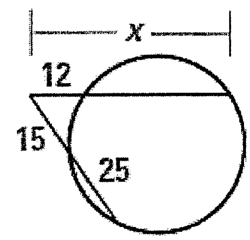
4.



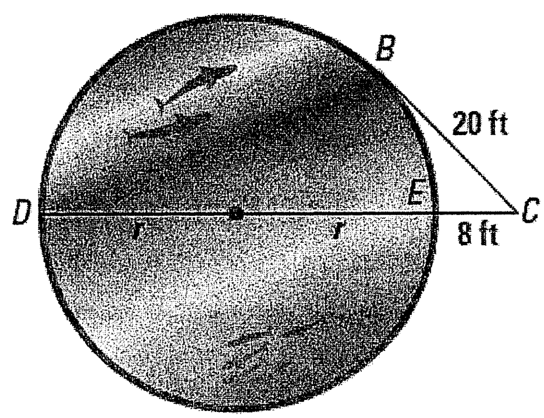
5.



6.



Example #5: You are standing at point C, about 8 feet from a circular aquarium tank. The distance from you to a point of tangency on the tank is about 20 feet. Estimate the radius of the tank.

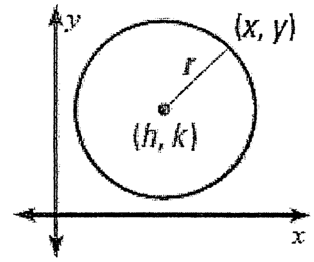


Chapter 10.7: Write and Graph Equations of Circle

Goal: Be able to write equations of circles in the coordinate plane.

The Standard Equation of a Circle:

The standard equation of a circle with center *(h, k)* and radius *r* is:



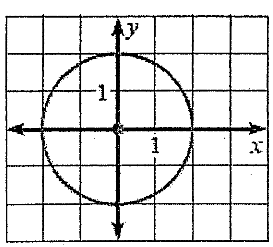
Example #1: Write the standard equation of the circle with center (-4, 0) and radius 7.1.

Example #2: Write the standard equation of the circle with center (0, -5) and radius 3.7.

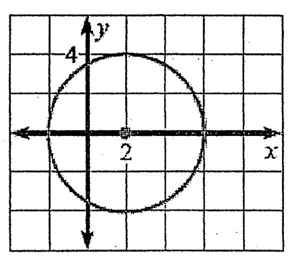
Example #3: Write the standard equation of the circle with center (-3, -5) and radius 6.1.

Example #4: Write the standard equation of the circles shown below.

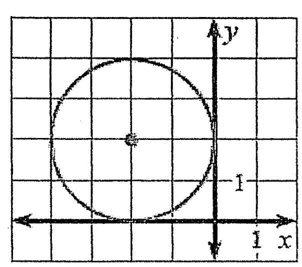
a.



b.



c.

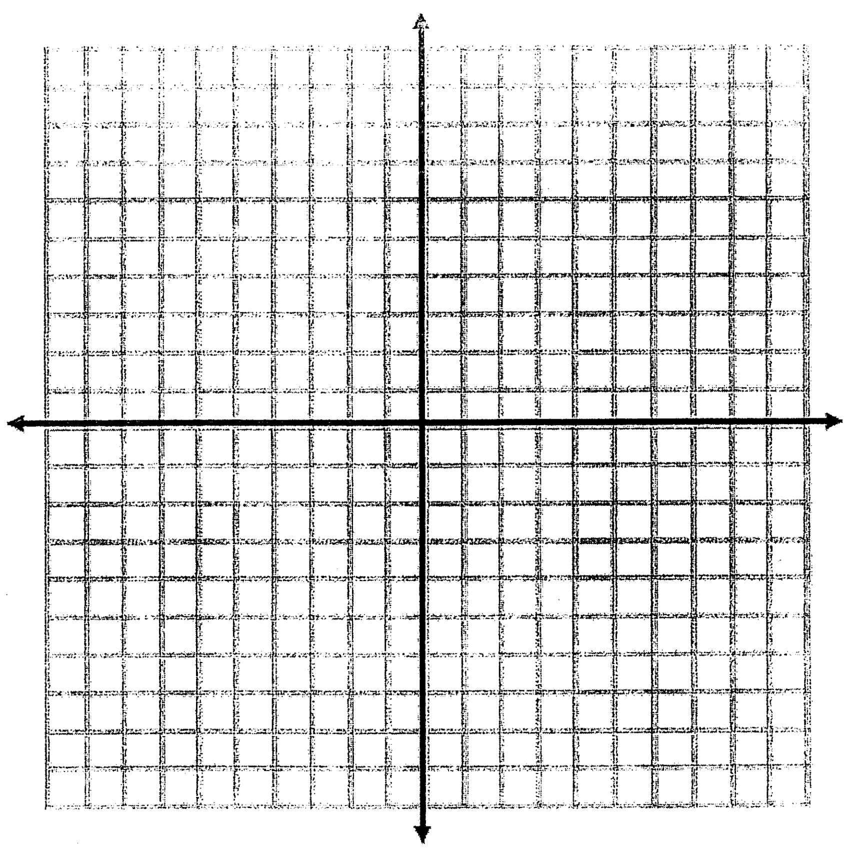


Example #5: The point (1, 2) is on a circle whose center is (5, -1). Write the standard equation of the circle.

Example #6: The point (-3, 4) is on a circle whose center is (-1, 2). Write the standard equation of the circle.

Example #7: Graph the following circles using their given equations.

a. $(x-2)^2+(y+3)^2=16$



b. $(x+5)^2+(y-5)^2=9$