Geometry

Ms. Linzmeier

Unit 11: Measuring Length and Area

Priority Standard:

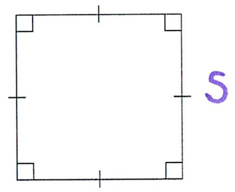
Unit 8 "I can" Statements:

1. I can find the area of triangles
2. I can find the area of parallelograms
3. I can find the area of squares
4. I can find the area of rectangles
5. I can find the area of trapezoids
6. I can find the area of kites
7. I can find the circumference of circles
8. I can find the arc measure and arc lengths of circles
9. I can find the area of circles
10. I can find the area of a sector of a circle
11. I can find the area of any regular polygon
12. I can find geometric probability using lengths and areas.

pg 723-725: 1-5 odd, 8, 10, 17-20 all, 25, 26, 38, 40

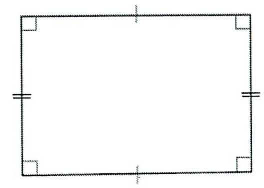
Chapter 11.1: Areas of Triangles and Parallelograms

Area of a Square Postulate (Postulate 24):



$A = s^2$

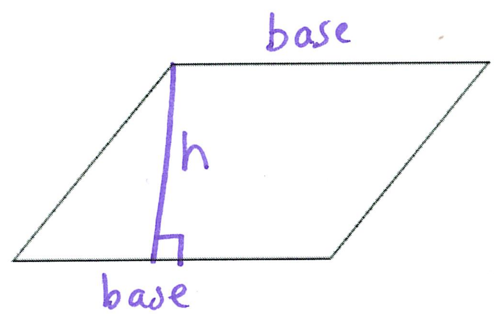
Area of a Rectangle (Theorem 11.1):



$A = b \cdot h$ or $A = l \cdot w$

Area of a Parallelogram (Theorem 11.2):

$A = b \cdot h$



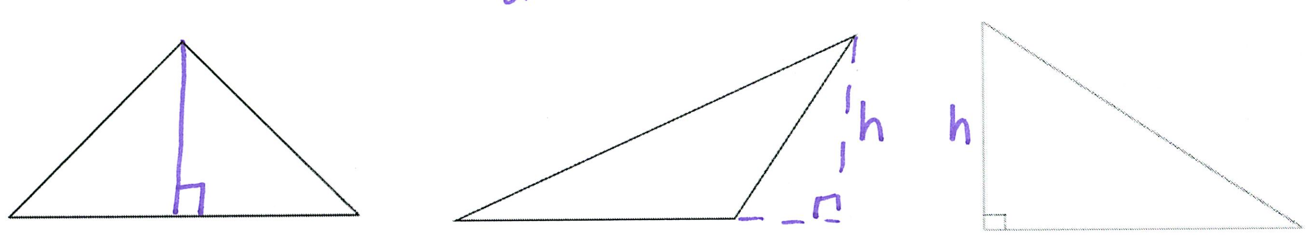
Bases of a Parallelogram: one of the parallel sides (either pair)

Height of a Parallelogram: is perpendicular distance between the bases

Area of a Triangle (Theorem 11.3):

$A = \frac{b \cdot h}{2}$ or $A = \frac{1}{2} b \cdot h$

* height is $\perp$ to base



Area Congruence Postulate (Postulate 25):

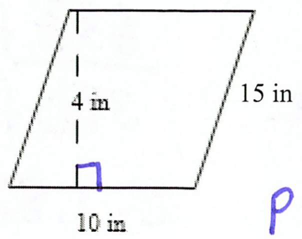
if two polygons are congruent, then their area is congruent.

Area Addition Postulate (Postulate 26):

the area of a region is the sum of the areas of its nonoverlapping parts

Example #1: Find the area and perimeter of the following figures.

a)



$A = 4 \cdot 10$

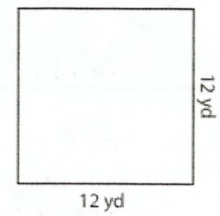
$\boxed{A = 40 in^2}$

$P = \underbrace{15 + 15}_{30} + \underbrace{10 + 10}_{20}$

$30 + 20$

$\boxed{P = 50 in}$

b)



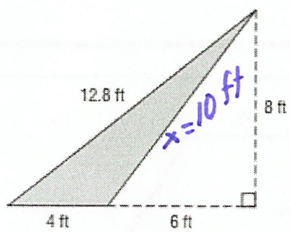
$A = 12^2$

$\boxed{A = 144 yd^2}$

$P = 12(4)$

$\boxed{P = 48 yds}$

c)



$6^2 + 8^2 = x^2$

$36 + 64 = x^2$

$\sqrt{100} = \sqrt{x^2}$

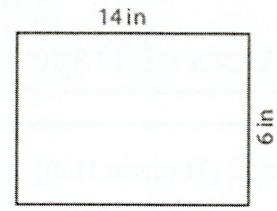
$10 = x$

$A = \frac{4 \cdot 8}{2} = \boxed{16 ft^2}$

$P = 12.8 + 4 + 10$

$\boxed{P = 26.8 ft}$

d)



$A = 14 \cdot 6$

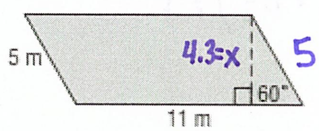
$\boxed{A = 84 in^2}$

$P = \underbrace{14 + 6}_{20} + \underbrace{14 + 6}_{20}$

$20 + 20$

$\boxed{P = 40 in}$

e)



$5 \left(\sin 60\right) = \left(\frac{x}{5}\right) 5$

$x = 4.3$

$A = 4.3 \cdot 11$

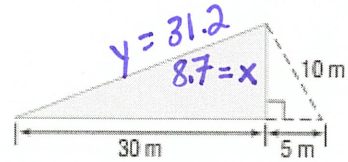
$\boxed{A \approx 47.3 m^2}$

$P = 11 + 11 + 5 + 5$

$= 22 + 10$

$\boxed{P = 32 m}$

f)



$P = 31.2$
$+30$
$+ 8.7$

$\boxed{P = 69.9 m}$

$A = \frac{30(8.7)}{2}$

$\boxed{A = 130.5 m^2}$

$x^2 + 5^2 = 10^2$

$-25 \quad -25$

$\sqrt{x^2} = \sqrt{75}$

$x \approx 8.7 m$

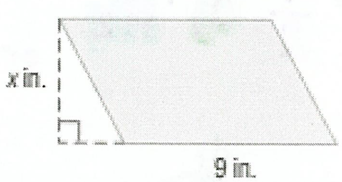
$30^2 + \overset{(x^2)}{75} = y^2$

$\sqrt{900 + 75} = \sqrt{y^2}$

$31.2 = y$

Example #2: Find x.

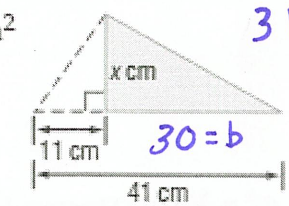
a) $A = 153$ in^2



$\frac{153}{9} = \frac{9x}{9}$

$\boxed{17 in = x}$

b) $A = 165$ cm^2



$165 = \frac{30x}{2}$

$\frac{165}{15} = \frac{15x}{15}$

$\boxed{x = 11 cm}$

Example #3: The base of a triangle is four times its height. The area of the triangle is 50 square inches. Find the base and height.

$b = 4h$

$h = \text{height}$

$A = 50 in^2$

$50 = \frac{4h \cdot h}{2}$

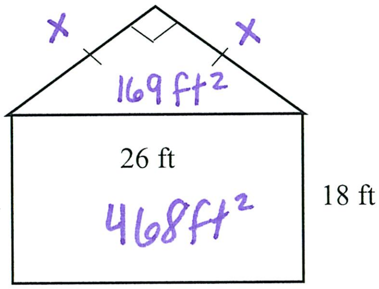
$\frac{50}{2} = \frac{2h^2}{2}$

$\sqrt{25} = \sqrt{h^2}$

$5 = h$

$\boxed{5 in}$

Example #4 Painting a barn: 1 gallon covers 350 square feet. How many gallons should you buy to paint the front side of the barn?



$A_{\triangle} = \frac{\sqrt{338}(\sqrt{338})}{2}$

$A_{\triangle} = \frac{338}{2} = 169$

$A_{\square} = 18 \cdot 26$

468

$A_{barn} = 169 + 468$
$= 637 \text{ ft}^2$

$x^2 + x^2 = 26^2$

$\frac{2x^2}{2} = \frac{676}{2}$

$\sqrt{x^2} = \sqrt{338}$

$x = \sqrt{338}$

$\frac{637 \text{ ft}^2}{350 \text{ ft}^2}$

≈ 1.82 gal

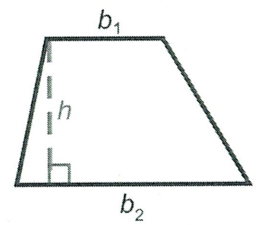
$\boxed{\text{Need 2 gallons}}$

Chapter 11.2: Areas of Trapezoids, Rhombuses and Kites

Area of a Trapezoid (Theorem 11.4):

The area of a trapezoid is one half the product of the height and the sum of the lengths of the bases

$A_{\text{trap}} = \frac{(b_1 + b_2)h}{2}$



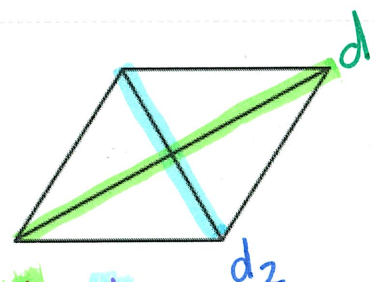
or

* The height of a trapezoid is the perpendicular distance between its bases

Area$_{\text{trap}}$ = $\frac{1}{2} \cdot (b_1 + b_2)h$

Area of a Rhombus (Theorem 11.5):

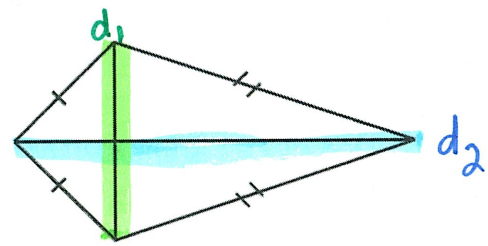
The area of a rhombus is one half the product of the lengths of its diagonals.



Area= $\frac{1}{2} d_1 \cdot d_2$ or $\frac{d_1 \cdot d_2}{2}$

Area of a Kite (Theorem 11.6):

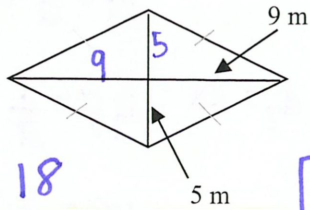
The area of a kite is one half the product of the lengths of its diagonals.



Area= $\frac{1}{2} \cdot d_1 \cdot d_2$ or $\frac{d_1 \cdot d_2}{2}$

Example #1: Find the area of the figures.

a)



$d_1 = 18$

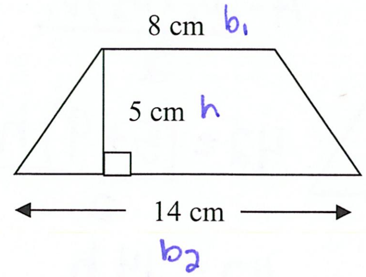
$d_2 = 10$

$A = \frac{1}{2} \cdot d_1 \cdot d_2$

$= \frac{1}{2}(18)(10)$

$\boxed{A_{\diamond} = 90 m^2}$

b)



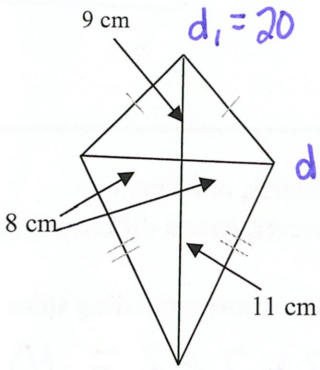
$A = \frac{(b_1 + b_2)h}{2}$

$= \frac{(8+14)5}{2}$

$= \frac{(22)5}{2}$

$\boxed{A_{\triangle} = 55 cm^2}$

c)



$d_1 = 20$

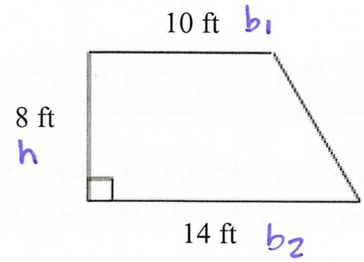
$d_2 = 16$

$A_{\diamond} = \frac{1}{2} d_1 \cdot d_2$

$= \frac{1}{2}(20)(16)$

$\boxed{A_{\diamond} = 160 cm^2}$

d)



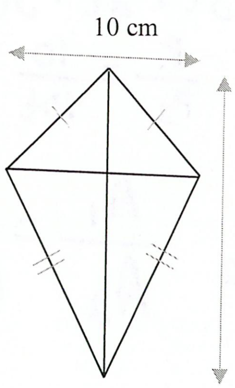
$A = \frac{(b_1 + b_2)h}{2}$

$= \frac{(10+14)8}{2}$

$= \frac{(24)8}{2}$

$\boxed{A_{\triangle} = 96 ft^2}$

e)

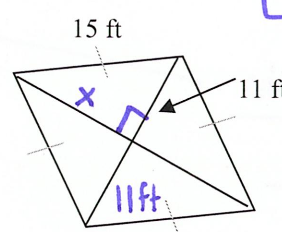


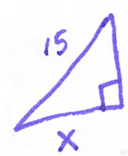
$A_{\diamond} = \frac{1}{2} d_1 \cdot d_2$

$= \frac{1}{2}(10)(16)$

$\boxed{A_{\diamond} = 80 in^2}$

f)





$11^2 + x^2 = 15^2$

$121 + x^2 = 225$

$-121 \quad -121$

$\sqrt{x^2} = \sqrt{104}$

$x \approx 10.2$

$d_1 = 22$

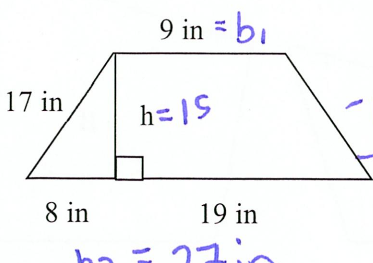
$d_2 = 10.2 + 10.2$

$d_2 = 20.4$

$A_{\triangle} = \frac{1}{2}(22)(20.4)$

$\boxed{A_{\triangle} \approx 224.4 ft^2}$

g)



$b_2 = 27 in$

$8^2 + h^2 = 17^2$

$-64 \quad -64$

$\sqrt{h^2} = \sqrt{225}$

$h = 15 in$

$A_{\triangle} = \frac{(b_1 + b_2)h}{2}$

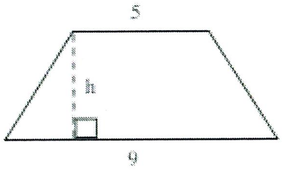
$= \frac{(9+27)15}{2}$

$= \frac{36(15)}{2}$

$\boxed{A_{\triangle} = 270 in^2}$

Example #2: Use the given information to find the value of *x*.

a) Area $=42$ ft^2



$A=\frac{(b_1+b_2)h}{2}$

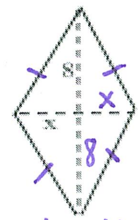
$42=\frac{(5+9)h}{2}$

$42=\frac{14h}{2}$

$\frac{42}{7}=\frac{7h}{7}$

$\boxed{h=6\text{ft}}$

b) Rhombus Area $=48$cm^2



$d_2=2x$

$d_1=16$

$A=\frac{1}{2}d_1\cdot d_2$

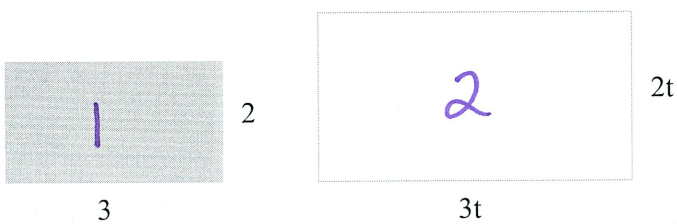
$48=\frac{1}{\cancel{2}}\frac{(\cancel{16}^{8})}{1}\frac{2x}{1}$

$\frac{48}{16}=\frac{16x}{16}$

$\boxed{3\text{ cm}=x}$

Chapter 11.3: Perimeter and Area of Similar Figures

Back in Chapter 6 you learned that if two polygons are similar, then the ratio of their perimeters, or of any two corresponding lengths, is equal to the ratio of their corresponding side lengths. Areas, however, have a different ratio.



side lengths

$\frac{3}{3t}=\frac{1}{t}$

$\frac{2}{2t}=\frac{1}{t}$

Ratio of Perimeters/corresponding sides

$P_{\boxed{1}}=2+3+2+3=10$

$P_{\boxed{2}}=2t+3t+2t+3t=10t$

$\frac{P_1}{P_2}=\frac{10}{10t}=\frac{1}{t}$

Ratio of Areas

$A_{\boxed{1}}=2\cdot3=6$

$A_{\boxed{2}}=2t\cdot3t=6t^2$

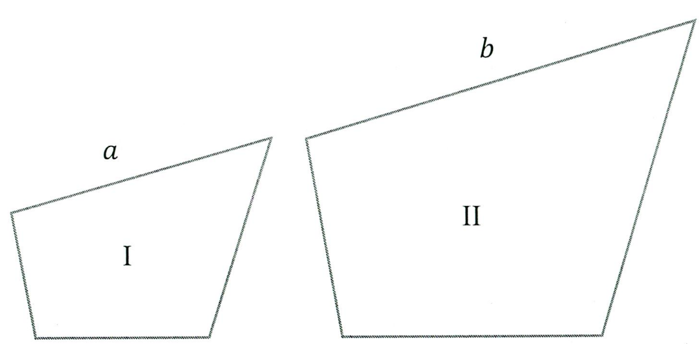
$\frac{A_1}{A_2}=\frac{6}{6t^2}=\frac{1}{t^2}$

Areas of Similar Polygons **(Theorem 11.7)**:

If two polygons are similar with the lengths of corresponding sides in the ratio of a: b, then the ratio of their area is a^2: b^2.

$$\frac{\text{Side length of Polygon 1}}{\text{Side length of Polygon II}}=\frac{a}{b}$$

$$\frac{\text{Area of Polygon 1}}{\text{Area of Polygon II}}=\frac{a^2}{b^2}$$

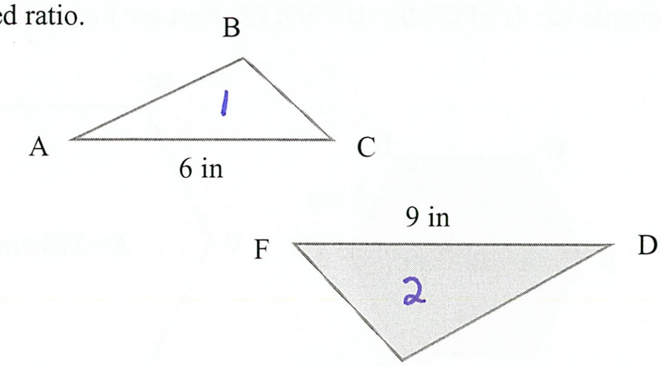


Polygon 1 ~ Polygon 2

Example #1: In the diagram, $\triangle ABC \sim \triangle DEF$. Find the indicated ratio.

a) Ratio (shaded to unshaded) of the perimeters

$$\frac{9}{6} = \boxed{\frac{3}{2}}$$



b) Ratio (shaded to unshaded) of the areas.

$$\frac{3^2}{2^2} = \boxed{\frac{9}{4}}$$

Example #2:

$\sqrt{a} : \sqrt{b}$

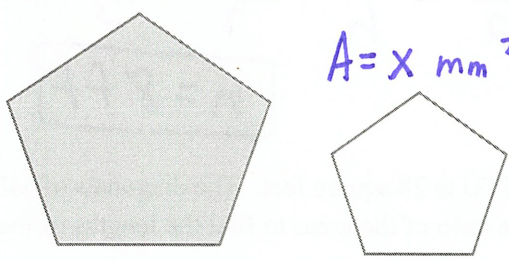
$a^2 : b^2$

Ratio of corresponding side lengths	Ratio of Perimeters	Ratio of Areas
5:8	5:8	25:64
4:7	4:7	16:49
13:6	13:6	169:36
66:18= 11:3	11:3	121:9

Example #3: Corresponding lengths in similar figures are given. Find the ratios (shaded to unshaded) of the perimeters and areas. Find the unknown area.

a) Shaded Area= 1024 mm^2

$A = x \text{ mm}^2$



24 mm   9 mm

$$R.O.P = \frac{24}{9} = \boxed{\frac{8}{3}}$$

$$R.O.A = \frac{8^2}{3^2} = \boxed{\frac{64}{9}}$$

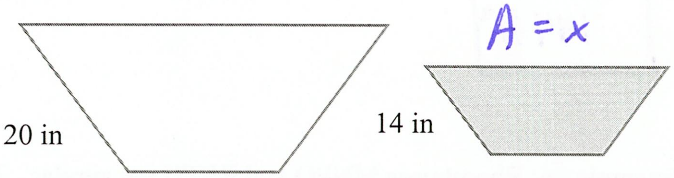
$$\frac{64}{9} = \frac{1024}{x}$$

$$\frac{64x}{64} = \frac{9216}{64}$$

$$\boxed{x = 144 \text{ mm}^2}$$

b) Unshaded Area= 400 in^2

$A = x$



20 in   14 in

$$R.O.P = \frac{14}{20} = \boxed{\frac{7}{10}}$$

$$R.O.A = \frac{7^2}{10^2} = \boxed{\frac{49}{100}}$$

$$\frac{49}{100} = \frac{x}{400}$$

$$\frac{100x}{100} = \frac{19600}{100}$$

$$\boxed{x = 196 \text{ in}^2}$$

Example #4:The ratio of the areas of two similar figures is given. Write the ratio of the lengths of corresponding sides.

a) Ratio of areas = 16:81

$\sqrt{16} : \sqrt{81}$

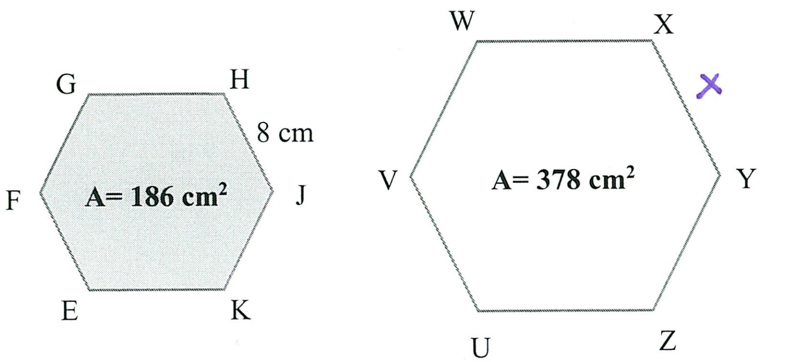
$\boxed{4:9}$

b) Ratio of areas =144:49

$\sqrt{144} : \sqrt{49}$

$\boxed{12:7}$

Example #5: If EFGHJK~UVWXYZ, then use the given area to find XY



$$\frac{186 \div 6}{378 \div 6} = \frac{31}{63}$$

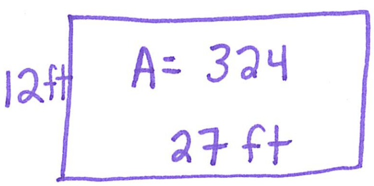
$$\frac{\sqrt{31}}{\sqrt{63}} = \frac{8}{x}$$

$$\frac{\sqrt{31}\,x}{\sqrt{31}} = \frac{63.5}{\sqrt{31}}$$

$$\frac{5.6}{7.9} = \frac{8}{x} \qquad \frac{5.6x}{5.6} = \frac{63.2}{5.6}$$

$x \approx 11.3$ cm or $\boxed{x \approx 11.4 \text{ cm}}$

Example #5: A large rectangular billboard is 12 feet high and 27 feet long. A smaller billboard is similar to the large billboard. The area of the smaller billboard is 144 square feet. Find the height of the smaller billboard.



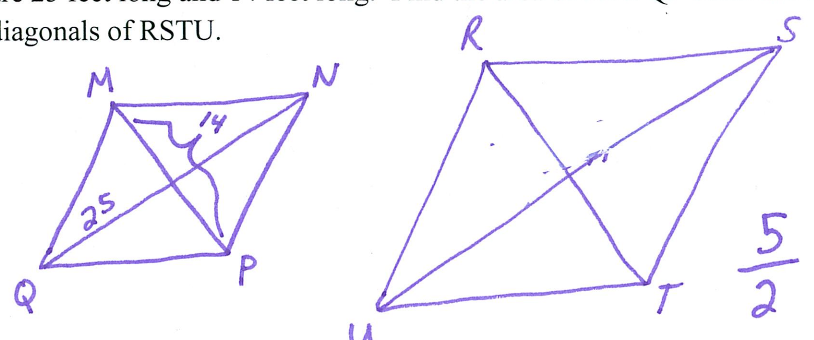
$$\text{R.O.A} = \frac{324 \div 12}{144 \div 12} = \frac{27}{12} = \frac{9}{4}$$

$$\text{R.O.P} = \frac{\sqrt{9}}{\sqrt{4}} = \frac{3}{2}$$

$$\frac{3}{2} = \frac{12}{h} \qquad \frac{3h}{3} = \frac{24}{3}$$

$$\boxed{h = 8 \text{ ft}}$$

Example #6: Rhombuses MNPQ and RSTU are similar. The area of RSTU is 28 square feet. The diagonals of MNPQ are 25 feet long and 14 feet long. Find the area of MNPQ. Then use the ratio of the areas to find the lengths of the diagonals of RSTU.



$$\text{R.O.S.L} = \frac{\sqrt{25}}{\sqrt{4}} = \frac{5}{2}$$

side lengths

$$\frac{5}{2} = \frac{14}{d_1} \qquad \frac{5}{2} = \frac{25}{d_2}$$

$$A = \frac{1}{2}(25)(14)$$

$A = 28 \text{ ft}^2$

$\boxed{A = 175 \text{ ft}^2}$ MNPQ

$$\frac{5d_1}{5} = \frac{28}{5} \qquad \frac{5d_2}{5} = \frac{50}{5}$$

$$\text{R.O.A} = \frac{175}{28} = \frac{25}{4}$$

$d_1 = 5.6$ ft $d_2 = 10$ ft

or

$d_1 \; 5\frac{3}{5}$